FIRSTRUN – Fiscal Rules and Strategies under Externalities and Uncertainties.
Funded by the Horizon 2020 Framework Programme of the European Union.
Project ID 649261.

FIRSTRUN Deliverable 3.2

One EMU Fiscal Policy for the EURO

Abstract

We build a Two-Country Open-Economy New-Keynesian DSGE model of a Currency Union to study the effects of fiscal policy coordination, by evaluating the stabilization properties of different fiscal policy scenarios. Our main findings are the following: a) a government spending rule that targets net exports rather than domestic output produces more stable dynamics, b) consolidating government budget constraints across countries and moving tax rates jointly provides greater stabilization than with separate budget constraints and independent tax rate movements, c) taxes on labour income are exponentially more distortionary than taxes on firm sales. These findings point out to possible policy prescriptions for the Eurozone: to coordinate fiscal policies by reducing international demand imbalances, either by stabilizing trade flows across countries or by creating some form of fiscal union or both, while avoiding the excessive use of labour taxes, in favour of sales taxes.

JEL classification: E62, E63, F42, F45, E12
Keywords: Fiscal Policy, International Policy Coordination, Monetary Union, New Keynesian

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Delivery date: 2016-08-17

One EMU Fiscal Policy for the EURO*

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August 17, 2016

Abstract

We build a Two-Country Open-Economy New-Keynesian DSGE model of a Currency Union to study the effects of fiscal policy coordination, by evaluating the stabilization properties of different fiscal policy scenarios. Our main findings are the following: a) a government spending rule that targets net exports rather than domestic output produces more stable dynamics, b) consolidating government budget constraints across countries and moving tax rates jointly provides greater stabilization than with separate budget constraints and independent tax rate movements, c) taxes on labour income are exponentially more distortionary than taxes on firm sales. These findings point out to possible policy prescriptions for the Eurozone: to coordinate fiscal policies by reducing international demand imbalances, either by stabilizing trade flows across countries or by creating some form of fiscal union or both, while avoiding the excessive use of labour taxes, in favour of sales taxes.

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*A special thanks to Pierpaolo Benigno for useful research guidance and to Giovanna Vallanti for helpful academic guidance. We would like to thank Konstantinos Tsinikos, Pietro Reichlin and seminar participants in LUISS Guido Carli for useful comments and suggestions. All errors are our own. We acknowledge financial support through the research project FIRSTRUN (Grant Agreement 649261) funded by the Horizon 2020 Framework Programme of the European Union.

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Contents

1 Introduction

Are there gains from fiscal policy coordination in a monetary union subject to alternative shocks? Does this create a scope for a centralized fiscal capacity in the EMU?

Given a single monetary policy in the European Economic and Monetary Union (EMU), country-specific shocks cannot be addressed through monetary policy, but must be balanced by country-specific fiscal policies. Whether this calls for coordination or not is a much debated issue, and has been typically investigated by looking at fiscal multipliers, as Farhi and Werning (2012*a*) finds a greater output multiplier if government spending is financed by a foreign country rather than the home country. This would create a scope for a central EMU budget, as centrally financed government spending has larger effects than nationally financed government spending. However, Farhi and Werning (2012*a*) uses a model without distortionary taxation, which brings to different dynamics. As emphasized in Forni, Gerali and Pisani (2010), a reduction in public spending followed by lower distortionary taxation can produce positive cross-country spillovers in the euro area.

We analyze the stabilization properties and the welfare implications of different scenarios for fiscal policy coordination in the EMU. The fiscal policy specifications are: uncoordinated fiscal policy (Pure Currency Union scenario), where each country chooses its government consumption, transfers and taxation; coordinated fiscal policy (Coordinated Currency Union scenario), where government consumption, transfers and taxation are chosen for each country by the union as a whole, with separate budget constraints; and fiscal union (Full Fiscal Union scenario), where government consumption, transfers and taxation are chosen for each country by the union as a whole, with a consolidated budget.

We analyze the welfare gains from coordination, considering whether there is a scope for a fiscal capacity in the EMU to address asymmetric shocks to member countries, as a shock-absorption mechanism, as addressed in Van Rompuy et al. (2012). This was mentioned also in the more recent Juncker et al. (2015), where a Fiscal Union is seen as a Euro area-wide macroeconomic stabilization tool, over and above national fiscal policies needed to cushion country-specific shocks, which is thought to be key in avoiding procyclical fiscal policies at all times. We define two welfare criteria and evaluate the welfare gains from a common macroeconomic stabilization function which can better deal with shocks that cannot be managed at the national level alone. We compare welfare under the Full Fiscal Union, the Coordinated Currency Union and the Pure Currency Union scenarios, bringing to policy conclusions for the proper macroeconomic management of a Currency Union.

Our analysis follows the open economy approach of Galí (2009), but in a two-country setting like in

Silveira (2006)[1]. Our model follows the specifications of Ferrero (2009), which adapts the optimal approach of Benigno and Woodford (2004) to monetary and fiscal policy in a cashless closed economy without capital, where there are only distortionary taxes as sources of government revenue, to a two-country open-economy Currency Union setting. We add home bias in consumption (or a degree of openness to international trade) and targeting rules for fiscal policy to the model in Ferrero (2009). The former allows for deviations from Purchasing Power Parity, while the latter is a fiscal policy rule. A similar model to our's is the Currency Union model of Benigno (2004), but without a fiscal authority and with money in the utility function.

As in Ferrero (2009), our model is structured to allow for spillovers from monetary to fiscal policy and viceversa, and from one country to another through country-specific fiscal policies. Nominal rigidities, in the form of staggered prices, generate real effects of monetary policy, while distortionary taxation generates non-Ricardian effects of fiscal policy. This framework allows to study the interaction between country-specific fiscal policies, where in the absence of the nominal exchange rate as an automatic stabilizer, fiscal policies influence each other through their effects on net exports and the terms of trade.

Our main findings are: that coordinating fiscal policies, by targeting net exports rather than output, produces more stable dynamics; that consolidating government budget constraints across countries and moving tax rates jointly provides greater stabilization than with separate budget constraints and independent tax rate movements; and that taxes on labour income are exponentially more distortionary than taxes on firm sales. These findings point out to possible policy prescriptions for the Eurozone: to coordinate fiscal policies by reducing international demand imbalances, either by stabilizing trade flows across countries or by creating some form of fiscal union or both, while avoiding the excessive use of labour taxes, in favour of sales taxes.

The remainder of the paper is structured as follows. Section 2 describes the general model and the fiscal policy scenarios of a Pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union. Section 3 presents the calibration of the parameters and steady state stances of the model to two groups of countries in the EMU. Section 4 describes two welfare criteria and provides welfare rankings of the different fiscal policy scenarios. Section 5 provides numerical simulations under different scenarios, comparing different degrees of fiscal policy coordination and alternative government financing schemes. Section 6 shows numerical simulations and welfare evaluations of the case for international goods as complements, rather

[1]The structure of Galí and Monacelli (2008) and Farhi and Werning (2012*b*) with a continuum of countries means that more variables will be exogenous, compared to a two-country model, and that a single country, being one of an infinite continuum, as specified in Galí (2009), does not influence any world variable. This means that all world variables must be exogenous and that it is harder to see the interaction among countries, so that international trade has no role because any expenditure on goods from any one country has a value of zero, being one of infinitely many composing the integral, as written in Galí (2009) that an integral of any variable over all countries is the same as an integral of the same variable over all countries except one. This poses questions on the validity of such a model and pushes us to prefer a two-country model instead, where the interactions among the two countries (or two groups of countries) are more evident and the dynamics are thus clearer.

than substitutes. Section 7 collects the main conclusions and provides possible extensions. Appendix A.1 provides all the equilibrium conditions of the model used for the simulations, while Appendix A.2 describes the steady state on which the model is calibrated.

2 A Two-Country Currency Union Model

The world economy is composed of two countries (or groups of countries), which form a Currency Union, the EMU. Both economies are assumed to share identical preferences, technology and market structure, but may be subject to different shocks, and have different price rigidities, initial conditions and fiscal stances. The two countries are indexed by H and F for *Home* and *Foreign*. We can think of country H as Germany and country F as the rest of the Eurozone. The world is populated by a continuum of infinitely-lived households of measure one, indexed by $i \in [0, 1]$. Each household owns a monopolistically competitive firm producing a differentiated good, indexed by $j \in [0, 1]$. The population on the segment $[0, h)$ belongs to country H while the population on the segment $[h, 1]$ belongs to country F. This means that the relative size of country H is $h \in [0, 1]$, while the relative size of country F is $1 - h$. This is true for both households and firms.

Firms set prices in a staggered fashion following Calvo (1983) and use only labour for production. There is no capital and no investment. Labour markets are competitive and internationally segmented, so that labour supply is country-wide and not firm-specific. All goods are tradable and the Law of One Price (LOP) holds for all single goods j. At the same time deviations from Purchasing Power Parity (PPP) may arise because of home bias in consumption. Financial markets are complete internationally, allowing households to trade a full set of one-period state-contingent claims across borders, other than purchase one-period risk-free bonds issued by the two countries' governments.

Each country has an independent Fiscal Authority, while the Currency Union shares a common Monetary Authority. The Central Bank, the ECB in the Eurozone, sets the nominal interest rate for the whole Currency Union following an Inflation Targeting regime, where the target is on union-wide CPI inflation. This assumption reflects the current functioning of the ECB, whose primary objective of price stability is formulated as a situation in which the one-year increase in the CPI for the Eurozone is less than, but close to, 2%. Fiscal policy is designed following the Fiscal Compact Rules, by imposing that the Government Debt-to-GDP ratio is about 60% and that countries must adopt a balanced budget law in their national legislation. Governments choose the level of government consumption and transfers, which are financed by distortionary taxes on labour income and firm sales and by short-term government bonds. In particular:

- In a Pure Currency Union scenario governments choose the level of government consumption for *domestic stabilization* purposes by setting it to target the output gap, financed by a mix of distortionary

tax rates, while balancing the budget. In this case Germany and the rest of the Eurozone manage fiscal policy independently without cooperating, because they only care about stabilizing their own domestic demand.

- In a Coordinated Currency Union scenario governments choose the level of local government consumption for *international stabilization* purposes by setting it to target the net exports gap, financed by a mix of distortionary tax rates, while balancing the budget. Here Germany and the rest of the Eurozone still manage fiscal policy independently, but decide to coordinate by stabilizing their trade flows.

- A Full Fiscal Union scenario uses a *consolidated budget constraint* to finance local government consumption for *international stabilization* purposes by setting it to target the net exports gap, financed by a mix of distortionary tax rates, while balancing the consolidated budget and varying equally the tax rates across countries, so as to use union-wide resources to finance the overall government expenditure. Here Germany and the rest of the Eurozone do not manage fiscal policy independently anymore and, while coordinating by stabilizing their trade flows, they also harmonize their tax rate movements to finance both countries' expenditures, as if there were only one country.

In what follows we denote variables referred to the *Foreign* country with a star (*) and, given symmetry between the two countries, we show the main equations only for country H, while we also show the equations for country F when they are different from those for country H.

2.1 Households

In each country there is a continuum of households, which gain utility from private consumption and disutility from labour, consume goods produced in both countries with home bias, supply labour to domestic firms and collect profits from those firms. Households can trade a complete set of one-period state-contingent claims across borders and purchase one-period risk-free bonds issued by the two countries' governments, subject to their budget constraint.

Each household in country H, indexed by $i \in [0, h)$, seeks to maximize the present-value utility[2]:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[\frac{(C_t^i)^{1-\sigma} - 1}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \right] \tag{2.1.1}$$

where $\beta \in [0, 1]$ is the common discount factor, which households use to discount future utility, σ is the

[2]We choose to specify additively separable period utility of the type with Constant Relative Risk Aversion (CRRA), so with constant elasticity of intertemporal substitution, and with constant elasticity of labour supply.

inverse of the elasticity of intertemporal substitution[3] (it is also the Coefficient of Relative Risk Aversion (CRRA)), φ is the inverse of the Frisch elasticity of labour supply[4], and ξ_t is a preference shock to *Home* households. This preference shock is assumed to follow the AR(1) process in logs:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \tag{2.1.2}$$

where $\rho_\xi \in [0,1]$ is a measure of persistence of the shock and ε_t is a zero mean white noise process. N_t^i denotes hours of labour supplied by households in country H. C_t^i is a composite index for private consumption defined by:

$$C_t^i \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t}^i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t}^i)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.1.3}$$

for households in country H, while the analogous index for households in country F, C_t^{*i}, is defined by:

$$C_t^{*i} \equiv \left[(1-\alpha^*)^{\frac{1}{\eta}} (C_{H,t}^{*i})^{\frac{\eta-1}{\eta}} + \alpha^{*\frac{1}{\eta}} (C_{F,t}^{*i})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.1.4}$$

where $C_{H,t}^i$ is an index of consumption of domestic goods for households in country H, given by the constant elasticity of substitution (CES) function (also known as Dixit and Stiglitz (1977) aggregator function):

$$C_{H,t}^i \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.1.5}$$

whereas, for households in country F the same index of consumption of domestic goods, $C_{H,t}^{*i}$, is given by:

$$C_{H,t}^{*i} \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{H,t}^{*i}(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.1.6}$$

where $j \in [0,1]$ denotes a single good variety of the continuum of differentiated goods produced in the world economy. $C_{F,t}^i$ is an index of consumption of imported goods for households in country H, given by the analogous CES function:

$$C_{F,t}^i \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{F,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.1.7}$$

[3]The elasticity of intertemporal substitution measures the responsiveness of consumption growth to changes in the real interest rate, which is the relative price of consumption between different dates, and is defined as the percent change in consumption growth divided by the percent change in the gross real interest rate.

[4]The Frisch elasticity of labour supply measures the extent to which labour supply responds to a change in the nominal wage, given a constant marginal utility of wealth, and is defined as the percent change in the supply of labour divided by the percent change in the nominal wage.

while the same index of consumption of imported goods for households in country F, $C^{*i}_{F,t}$, is given by:

$$C^{*i}_{F,t} \equiv \left(\left(\frac{1}{h} \right)^{\frac{1}{\varepsilon}} \int_0^h C^{*i}_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.1.8}$$

The parameter $\varepsilon > 1$ measures the elasticity of substitution between varieties produced within a given country. The parameter $\eta > 0$ measures the substitutability between domestic and foreign goods (international trade elasticity). The parameter $\alpha \in [0,1]$ is a measure of openness of the *Home* economy to international trade. Equivalently $(1-\alpha)$ is a measure of the degree of home bias in consumption in country H. When α tends to zero the share of foreign goods in domestic consumption vanishes and the country ends up in autarky, consuming only domestic goods. If $1-\alpha > h$ there is home bias in consumption in country H, because the share of consumption of domestic goods is greater than the share of production of domestic goods. The same applies to the *Foreign* parameter of openness to international trade $\alpha^* \in [0,1]$ for country F, except for the fact that if $1-\alpha^* > 1-h$ there is home bias in consumption in country F, because the share of consumption of domestic goods in country F is greater than the share of production of domestic goods in country F.

Households in country H maximize their present-value utility, equation 2.1.1, subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}(j) C^i_{H,t}(j) \, dj + \int_h^1 P_{F,t}(j) C^i_{F,t}(j) \, dj + D^i_t + B^i_t \leq \frac{D^i_{t-1}}{\mathcal{Q}_{t-1,t}} + B^i_{t-1}(1+i_{t-1}) + (1-\tau^w_t) W_t N^i_t + T^i_t + \Gamma^i_t \tag{2.1.9}$$

for $t = 0, 1, 2, \ldots$, where $P_{H,t}(j)$ is the price of domestic variety j, $P_{F,t}(j)$ is the price of variety j imported from country F, D^i_{t-1} is the portfolio of state-contingent claims purchased by the household in period $t-1$, $\mathcal{Q}_{t-1,t}$ is the stochastic discount factor, which is the same for households in both countries and represents the price of state-contingent claims or equivalently the inverse of the gross return on state-contingent claims, B^i_{t-1} are risk-free government bonds (of either or both governments) purchased by the household in period $t-1$, i_{t-1} is the nominal interest rate set by the central bank in period $t-1$, which is also the net return on these government bonds, W_t is the nominal wage for households in country H, T^i_t denotes lump-sum transfers from the government to households, Γ^i_t denotes the share of profits net of taxes to households from ownership of firms and $\tau^w_t \in [0,1]$ is a marginal tax rate on labour income paid by households to the government.

All variables are expressed in units of the union's currency. Last but not least, households in country H are subject to the following solvency constraint, for all t, that prevents them from engaging in Ponzi-schemes:

$$\lim_{T \to \infty} E_t \left\{ \mathcal{Q}_{t,T} D^i_T \right\} \geq 0 \tag{2.1.10}$$

Aggregating the intratemporal optimality condition yields the aggregate labour supply equation for households in country H:

$$N_t = (h)^{1+\frac{\sigma}{\varphi}}(C_t)^{-\frac{\sigma}{\varphi}}\left[(1-\tau_t^w)\frac{W_t}{P_t}\right]^{\frac{1}{\varphi}} \tag{2.1.11}$$

where N_t is aggregate labour supply and C_t is aggregate consumption for households in country H, defined by:

$$N_t \equiv \int_0^h N_t^i\, di = hN_t^i \qquad C_t \equiv \int_0^h C_t^i\, di = hC_t^i \tag{2.1.12}$$

while aggregating the intertemporal optimality condition for households in country H, taking conditional expectations and using the no-arbitrage condition between government bonds and state-contingent claims, yields:

$$\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} = \beta E_t\left\{\frac{\xi_{t+1}}{\xi_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{\Pi_{t+1}}\right\} \tag{2.1.13}$$

where $\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\}$ is the price of a one-period riskless government bond paying off one unit of the union's currency in $t+1$ and $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is gross CPI inflation in country H.

Aggregating the budget constraints of households in country H and considering that in optimality they hold with equality yields:

$$P_tC_t + D_t + B_t = (1+i_{t-1})(D_{t-1}+B_{t-1}) + (1-\tau_t^w)W_tN_t + T_t + \Gamma_t \tag{2.1.14}$$

where aggregate contingent claims, aggregate bonds, aggregate transfers and aggregate profits are defined analogously to aggregate consumption and labour.

The Consumer Price Index (CPI) for country H is given by:

$$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.1.15}$$

while the Consumer Price Index (CPI) for country F is given by:

$$P_t^* \equiv \left[(1-\alpha^*)(P_{H,t}^*)^{1-\eta} + \alpha^*(P_{F,t}^*)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{2.1.16}$$

where $P_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country H and $P_{F,t}$ is a price index for goods imported from country F, respectively defined by:

$$P_{H,t} \equiv \left(\frac{1}{h}\int_0^h P_{H,t}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}} \tag{2.1.17}$$

$$P_{F,t} \equiv \left(\frac{1}{1-h} \int_h^1 P_{F,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{2.1.18}$$

while $P^*_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country F and $P^*_{F,t}$ is a price index for goods imported from country H, respectively defined by:

$$P^*_{H,t} \equiv \left(\frac{1}{1-h} \int_h^1 P^*_{H,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{2.1.19}$$

$$P^*_{F,t} \equiv \left(\frac{1}{h} \int_0^h P^*_{F,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{2.1.20}$$

Since one-period state-contingent claims can be traded freely between households within and across borders, they are in zero international net supply, so that the market clearing condition for these assets in every period t is consequently given by:

$$\int_0^h D^i_t \, di + \int_h^1 D^{*i}_t \, di = hD^i_t + (1-h)D^{*i}_t = D_t + D^*_t = 0 \tag{2.1.21}$$

2.2 International Identities and Assumptions

Several international identities and assumptions need to be spelled out in order to link the *Home* economy to the *Foreign* one and to be able to close the model.

The *terms of trade* are defined as the price of foreign goods in terms of home goods, for households in country H and in country F, and are given respectively by:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} \qquad \text{and} \qquad \mathcal{S}^*_t \equiv \frac{P^*_{F,t}}{P^*_{H,t}} \tag{2.2.1}$$

Although deviations from *Purchasing Power Parity (PPP)* may arise because of home bias in consumption, we assume that the *Law of One Price (LOP)* holds for every single good j, which implies:

$$P_{H,t}(j) = P^*_{F,t}(j) \qquad \text{and} \qquad P_{F,t}(j) = P^*_{H,t}(j) \tag{2.2.2}$$

for all $j \in [0,1]$, where $P_{H,t}(j)$ (or $P_{F,t}(j)$ for goods imported from country F) is the price of good j in country H and $P^*_{F,t}(j)$ (or $P^*_{H,t}(j)$ for goods produced in country F) is the price of good j in country F in terms of the union's currency. Plugging the previous expressions into the definitions of $P_{H,t}$ and $P_{F,t}$ respectively yields:

$$P_{H,t} = P^*_{F,t} \qquad \text{and} \qquad P_{F,t} = P^*_{H,t} \tag{2.2.3}$$

Combining the previous result with the definition of the terms of trade for countries H and F yields:

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}} = \frac{P^*_{H,t}}{P^*_{F,t}} = \frac{1}{\mathcal{S}^*_t} \tag{2.2.4}$$

The relationship between PPI inflation and CPI inflation in country H is given by:

$$\Pi_t = \Pi_{H,t} \left[\frac{1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}}{1 - \alpha + \alpha(\mathcal{S}_{t-1})^{1-\eta}} \right]^{\frac{1}{1-\eta}} \tag{2.2.5}$$

while dividing the terms of trade in period t by the terms of trade in period $t-1$ yields a relationship showing the evolution of the terms of trade over time:

$$\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}} = \frac{\Pi^*_{H,t}}{\Pi_{H,t}} \implies \mathcal{S}_t = \frac{\Pi^*_{H,t}}{\Pi_{H,t}} \mathcal{S}_{t-1} \tag{2.2.6}$$

as a function of PPI inflation in both countries H and F.

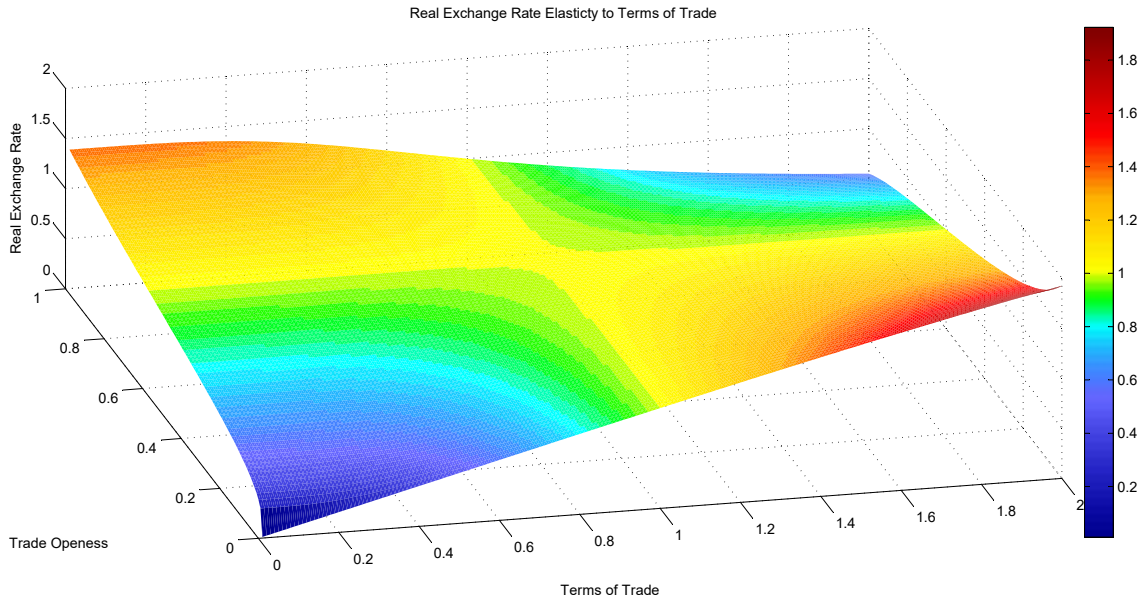
The *Real Exchange Rate* between the *Home* country and country F is the ratio of the two countries' CPIs, expressed both in terms of the union's currency, and is defined by:

$$Q_t \equiv \frac{P^*_t}{P_t} = \mathcal{S}_t \left[\frac{1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}}{1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}} \right]^{\frac{1}{1-\eta}} \tag{2.2.7}$$

where the difference between the real exchange rate and the terms of trade is given by the degree of openness of the two countries and the international trade elasticity. If the countries both have complete home bias ($\alpha = \alpha^* = 0$), then they are in autarky and the real exchange rate is exactly equal to the terms of trade, because the CPI and PPI are the same in each country.

The home bias in consumption generates a gap between the relative production price indices and the relative consumption price indices based on the different composition of the households' consumption basket in the two countries. Hence, the dynamics of the real exchange rate follow the dynamics of the terms of trade in a non-linear way, depending on the calibration of the degree of home bias. As Figure 1 shows, the real exchange rate increases as the terms of trade increase if the degree of openness of country H is less than the size of country F ($\alpha < 1 - h = 0.6$), which is the case for our calibration ($\alpha = 0.52$), while the real exchange rate decreases when the terms of trade increase if the degree of openness of country H is more than the size of country F. These two conditions imply respectively that the degree of openness of country F is less than the size of country H ($\alpha^* < h = 0.4$) for the real exchange rate to increase as the terms of trade increase, and that the degree of openness of country F is more than the size of country H for the real exchange rate to decrease as the terms of trade increase.

Figure 1: Elasticity of the Real Exchange Rate to the Terms of Trade as a function of Trade Openness



2.3 Net Exports, Net Foreign Assets and the Balance of Payments

Net Exports are defined as domestic production minus domestic consumption, which is equal to exports minus imports, and for country H are given in real terms (divided by $P_{H,t}$) by:

$$\widetilde{NX}_t = Y_t - \frac{P_t}{P_{H,t}} C_t - G_t = Y_t - \left[1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C_t - G_t \tag{2.3.1}$$

where net exports are shown to be a function of the country's degree of openness and the terms of trade, other than domestic production and public and private domestic consumption.

Since exports for country H are imports for country F and viceversa, then net exports are in zero international net supply. In real terms: $\widetilde{NX}_t + S_t \widetilde{NX}_t^* = 0$.

Net Foreign Assets are given by the sum of private and public assets held abroad, and for country H are

given in real terms (divided by $P_{H,t}$) by:

$$\widetilde{NFA}_t \equiv \tilde{D}_t + \tilde{B}_t - \tilde{B}_t^G \tag{2.3.2}$$

Since foreign assets for country H are domestic assets for country F, then net foreign assets are in zero international net supply. In real terms: $\widetilde{NFA}_t + S_t \widetilde{NFA}_t^* = 0$.

The *Balance of Payments* is given by net exports plus interest accrued on net foreign assets and is given in real terms (divided by $P_{H,t}$) by:

$$\widetilde{BP}_t \equiv \widetilde{NX}_t + i_{t-1} \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} \tag{2.3.3}$$

Combining the relationships between net foreign assets and net exports in the two countries shows that also the balance of payments of the two countries are in zero international net supply: $\widetilde{BP}_t + S_t \widetilde{BP}_t^* = 0$.

From the households' budget constraint, substituting in firm profits, labour income, the expression for transfers backed out from the government budget constraint, the definitions of net exports and net foreign assets and the definition of the balance of payments, yields the following relationship between net foreign assets and the balance of payments in real terms (divided by $P_{H,t}$) for country H:

$$\widetilde{NFA}_t = (1 + i_{t-1}) \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{NX}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t \tag{2.3.4}$$

which shows that the balance of payments is equal to the variation of net foreign assets over one period. Notice that all variables with a tilde ($\tilde{}$) are in real terms (divided by $P_{H,t}$).

2.4 International Risk-Sharing

The no-arbitrage condition for state-contingent claims and the assumption of complete markets implies that the price of the state-contingent claims must be the same for households in both countries. This in turn implies that we can link the consumption of households in the two countries by equating their Euler Equations through their Stochastic Discount Factor, which yields an international risk-sharing condition.

The risk sharing condition linking consumption of households in country H to consumption of households in country F, through their Euler Equations, is given by:

$$C_{t+1}^i = \left(\frac{\xi_{t+1}}{\xi_t} \frac{\xi_t^*}{\xi_{t+1}^*} \frac{Q_{t+1}}{Q_t} \right)^{\frac{1}{\sigma}} \frac{C_t^i}{C_t^{*i}} C_{t+1}^{*i} \tag{2.4.1}$$

By repeated substitution of *Home* consumption backwards in time and moving back one period, the previous equation reduces to one linking the consumption of households in the two countries as a function of initial

conditions, the real exchange rate and preference shocks:

$$C_t^i = \left(\frac{\xi_0^*}{\xi_0}\frac{1}{Q_0}\right)^{\frac{1}{\sigma}} \left(\frac{C_0^i}{C_0^{*i}}\right) \left(\frac{\xi_t}{\xi_t^*}Q_t\right)^{\frac{1}{\sigma}} C_t^{*i} \tag{2.4.2}$$

By assuming symmetric initial conditions for households in countries H and F, the previous expression reduces to:

$$C_t^i = \left(\frac{\xi_t}{\xi_t^*}Q_t\right)^{\frac{1}{\sigma}} C_t^{*i} \tag{2.4.3}$$

where household consumption in the two countries differs only in the presence of asymmetric preference shocks and of deviations from purchasing power parity (when the real exchange rate is different from one).

Aggregating the previous condition across households in each country yields:

$$C_t = \frac{h}{1-h}\left(\frac{\xi_t}{\xi_t^*}Q_t\right)^{\frac{1}{\sigma}} C_t^* \tag{2.4.4}$$

where C_t and C_t^* are aggregate consumption for households in countries H and F respectively and the difference in aggregate consumption across countries is also given by the relative size of the two countries.

Substituting the real exchange rate with equation 2.2.7 yields an international risk-sharing condition linking aggregate consumption in the two countries as a function of the terms of trade, the degree of openness of the two countries and the international trade elasticity, other than preference shocks and country size:

$$C_t = \frac{h}{1-h}\left[\frac{\xi_t}{\xi_t^*}\mathcal{S}_t\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1}{1-\eta}}\right]^{\frac{1}{\sigma}} C_t^* \tag{2.4.5}$$

2.5 Firms

In country H there is a continuum of firms indexed by $j \in [0, h)$ each producing a differentiated good with the same technology represented by the following production function:

$$Y_t(j) = A_t N_t(j) \tag{2.5.1}$$

where A_t represents the level of technology in country H, which evolves exogenously over time following the AR(1) process in logs:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \tag{2.5.2}$$

where $\rho_a \in [0, 1]$ is a measure of persistence of the shock and ε_t is a zero mean white noise process.

From the production function we can derive labour demand for individual firms in country H and the

respective nominal and real marginal costs of production, which are equal across firms in each country and are given by:

$$N_t(j) = \frac{Y_t(j)}{A_t} \implies MC_t^n = \frac{W_t}{A_t} \implies MC_t = \frac{W_t}{A_t P_{H,t}} \tag{2.5.3}$$

Aggregating individual labour demand across firms in each country yields the aggregate labour demand for country H:

$$N_t \equiv \int_0^h N_t(j)\, dj = \int_0^h \frac{Y_t(j)}{A_t}\, dj = \frac{Y_t}{A_t} \int_0^h \frac{1}{h} \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} dj = \frac{Y_t}{A_t} d_t \tag{2.5.4}$$

where Y_t and Y_t^* are aggregate output in countries H and F, respectively given by:

$$Y_t \equiv \left(\left(\frac{1}{h} \right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad Y_t^* \equiv \left(\left(\frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 Y_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.5.5}$$

and where the terms:

$$d_t \equiv \int_0^h \frac{1}{h} \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} dj \qquad \text{and} \qquad d_t^* \equiv \int_h^1 \frac{1}{1-h} \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} dj \tag{2.5.6}$$

represent relative price dispersion across firms in each country. In steady state and in a flexible price equilibrium these relative price dispersions are equal to one.

Aggregating over all $j \in [0, h)$ firm j's period t profits net of taxes in country H, substituting in labour demand, marginal costs, the demand function for output, using the definition of $P_{H,t}$, and substituting in price dispersion yields aggregate profits net of taxes in country H:

$$\Gamma_t = (1 - \tau_t^s) P_{H,t} Y_t - P_{H,t} MC_t Y_t d_t = P_{H,t} Y_t (1 - \tau_t^s - MC_t d_t) \tag{2.5.7}$$

where τ_t^s is the marginal tax rate on firm sales in country H.

Following Calvo (1983), each firm in country H may reset its price with probability $1 - \theta$ in any given period. Thus, each period a fraction $1 - \theta$ of randomly selected firms reset their price, while a fraction θ keep their prices unchanged. As a result, the average duration of a price in country H is given by $(1 - \theta)^{-1}$, and θ can be seen as a natural index of price stickiness for country H. In country F each firm may reset its price with probability $1 - \theta^*$ in any given period. This allows for the two countries to have different degrees of price rigidity.

A firm in country H re-optimizing in period t will choose the price $\bar{P}_{H,t}$ that maximizes the current market value of the profits net of taxes generated while that price remains effective. Formally, it solves the

problem:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \mathcal{Q}_{t,t+k} Y_{t+k|t}(j) \left[(1-\tau^s_{t+k}) \bar{P}_{H,t} - MC^n_{t+k} \right] \right\} \tag{2.5.8}$$

subject to the sequence of demand constraints[5]:

$$Y_{t+k|t}(j) = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \frac{Y_{t+k}}{h} \tag{2.5.9}$$

for $k = 0, 1, 2, \ldots$, where $\mathcal{Q}_{t,t+k}$ is the households' stochastic discount factor in country H for discounting k-period ahead nominal payoffs from ownership of firms, defined by:

$$\mathcal{Q}_{t,t+k} = \beta^k \frac{\xi_{t+k}}{\xi_t} \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \tag{2.5.10}$$

for $k = 0, 1, 2, \ldots$, and where $Y_{t+k|t}(j)$ is the output in period $t+k$ for firm j which last reset its price in period t.

The optimal price chosen by firms in country H can be expressed as a function of only aggregate variables:

$$\bar{P}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k} (C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} MC^n_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k} (C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} (1-\tau^s_{t+k}) \right\}} \tag{2.5.11}$$

Notice that in the zero inflation steady state and in the flexible price equilibrium the previous equation simplifies to:

$$\bar{P}_H = \frac{\varepsilon}{(\varepsilon - 1)(1 - \tau^s)} MC^n \tag{2.5.12}$$

where MC^n is the nominal marginal cost in steady state and in the flexible price equilibrium in country H, and where the optimal price is shown to be set as a markup over nominal marginal costs.

2.6 Central Bank and Monetary Policy

The only central bank in the currency union, the ECB in the Eurozone, sets monetary policy by choosing the nominal interest rate to target *union-wide* inflation through a Taylor rule. We assume that the ECB cares only about inflation, as price stability is its primary objective.

Monetary policy follows an Inflation Targeting regime of the kind:

$$\beta(1 + i_t) = \left(\frac{\Pi^U_t}{\Pi^U} \right)^{\phi_\pi (1-\rho_i)} [\beta(1 + i_{t-1})]^{\rho_i} \tag{2.6.1}$$

[5]The derivation of the demand function for firms is much like the derivation of the demand function for consumption goods, except for the timing of price setting, which implies that $P_{H,t+k}(j) = \bar{P}_{H,t}(j)$ with probability θ^k for $k = 0, 1, 2, \ldots$, and the fact that all firms are the same and so they set the same price in any given period, which allows us to drop the j index.

where union-wide inflation is defined as the population-weighted geometric average of the CPI inflation in the two countries:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \tag{2.6.2}$$

while variables without subscripts t denote their respective steady state levels, ϕ_π represents the responsiveness of the interest rate to inflation and ρ_i is a measure of the persistence of the interest rate over time (interest rate smoothing).

2.7 Government and Fiscal Policy in a Pure Currency Union

In a Pure Currency Union (uncoordinated fiscal policy) each government chooses the amount of government consumption and transfers for domestic stabilization purposes, financed by marginal tax rates on labour income and firm sales and by short-term government bonds. In this case Germany and the rest of the Eurozone manage fiscal policy independently without cooperating, because they only care about stabilizing their own domestic demand.

In country H the government finances a stream of public consumption G_t and transfers T_t subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}(j) G_t(j)\, dj + \int_0^h T_t^i\, di + B_{t-1}^G (1 + i_{t-1}) = B_t^G + \tau_t^s P_{H,t} Y_t + \int_0^h \tau_t^w W_t N_t^i\, di \tag{2.7.1}$$

where the right hand side represents government income from taxation and newly issued government bonds, while the left hand side represents total government spending on consumption and transfers, and on government bonds due at the end of period t, including interest. B_t^G are government bonds issued by country H in period t. Government consumption, G_t, is given by the following CES function, just like equation 2.5.9 for the demand function for firms, where we assume that the government purchases only goods produced domestically (complete home bias):

$$G_t \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h G_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.7.2}$$

Integrating the government budget constraint and dividing it by $P_{H,t}$ yields the aggregate government budget constraint in real terms:

$$G_t + \tilde{T}_t + i_{t-1} \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{2.7.3}$$

where variables with a tilde ($\tilde{}$) are in real terms (divided by $P_{H,t}$), and where the left hand side represents

current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of government debt.

Fiscal policy in country H chooses government consumption to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{2.7.4}$$

while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{2.7.5}$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ through the following tax rule[6]:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{2.7.6}$$

where $\psi_y \geq 0$ represents the responsiveness of government consumption to variations of the output gap, $\rho_g \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while variables without subscript t represent their respective steady state level.

Since government bonds are traded freely within and across borders without frictions and are perfectly substitutable because they offer the same return, the total amount of bonds held by households in both countries must equal the total amount of bonds issued by the two countries' governments, so that the market clearing condition for these assets in every period t is given in real terms (divided by $P_{H,t}$) by:

$$\tilde{B}_t + S_t \tilde{B}_t^* = \tilde{B}_t^G + S_t \tilde{B}_t^{*G} \tag{2.7.7}$$

[6] If the overall tax rate is defined as:

$$\tau_t^o \equiv \tau_t^s + \tau_t^w$$

then the variation of the tax rates on labour income and on firm sales will be given respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ of the variation of the overall tax rate in the following way:

$$(\tau_t^w - \tau^w) \equiv \gamma(\tau_t^o - \tau^o)$$
$$(\tau_t^s - \tau^s) \equiv (1-\gamma)(\tau_t^o - \tau^o)$$

which implies the tax rule in the text.

2.8 Government and Fiscal Policy in a Coordinated Currency Union

If the Governments of the two countries choose to coordinate, they will use their fiscal instruments to target a common objective, while maintaining independent budget constraints. Instead of using government consumption to stabilize the domestic output gap countercyclically, we assume that they use the same fiscal instrument to stabilize the net exports gap procyclically. This represents the act of coordinating their policies on a common objective, which depends on the interactions between the two economies, for international rather than domestic stabilization purposes. The budget constraints of the two fiscal authorities instead remain unmodified. Here Germany and the rest of the Eurozone still manage fiscal policy independently, but decide to coordinate by stabilizing their trade flows.

We choose the net exports gap as a common objective because one of the main concerns emerging in the euro area in the past years is the deep asymmetry between core countries, such as Germany, running current account surpluses and periphery countries running current account deficits. In particular, current account imbalances in the euro area have grown considerably, creating concerns about economic growth and the sustainability of a common currency[7].

Fiscal policy in country H chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{2.8.1}$$

while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{B}^G_t = \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} \tag{2.8.2}$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ through the following tax rule:

$$\gamma(\tau^s_t - \tau^s) = (1-\gamma)(\tau^w_t - \tau^w) \tag{2.8.3}$$

where $\psi_{nx} \geq 0$ represents the responsiveness of government consumption to variations of the real net exports gap, $\rho_g \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while variables without subscript t represent their respective steady state level.

[7]For references on current account imbalances in the euro area see Kollmann et al. (2014) and Schmitz and Von Hagen (2011), while we follow Hjortsø (2012) in our idea to coordinate fiscal policy by reducing international demand imbalances.

2.9 Government and Fiscal Policy in a Full Fiscal Union

If instead of considering two fiscal authorities managing fiscal policy independently, one for each country, or coordinating their policies, but with two separate budget constraints, we consider only one fiscal authority managing fiscal policy for both countries at the same time in a coordinated way and with a consolidated budget constraint, then we can think of it as an extreme case of fiscal policy coordination and call it a Full Fiscal Union. Here Germany and the rest of the Eurozone do not manage fiscal policy independently anymore and, while coordinating by stabilizing their trade flows, they also harmonize their tax rate movements to finance both countries' expenditures together, as if there were only one country.

A Full Fiscal Union uses local government spending to manage fiscal policy at the union level with a consolidated budget constraint. The Fiscal Union finances streams of local public consumption, G_t and G_t^*, and transfers, T_t and T_t^*, subject to the consolidated budget constraint of the two national fiscal authorities given in real terms (for country H) by:

$$G_t + \tilde{T}_t + S_t(G_t^* + \tilde{T}_t^*) + i_{t-1}\frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} = (\tau_t^s + \tau_t^w MC_t d_t)Y_t + (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*)S_t Y_t^* + \tilde{\mathcal{B}}_t^G - \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \quad (2.9.1)$$

where variables with a tilde ($\tilde{}$) are in real terms (divided by $P_{H,t}$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of overall government debt, which is given by:

$$\tilde{\mathcal{B}}_t^G = \tilde{B}_t^G + S_t \tilde{B}_t^{*G} \quad (2.9.2)$$

Union-wide fiscal policy chooses government consumption in each country to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule for country H:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \quad (2.9.3)$$

while keeping real transfers constant in each country and balancing the overall budget:

$$\tilde{T}_t = \tilde{T} \qquad \text{and} \qquad \tilde{\mathcal{B}}_t^G = \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \implies \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = S_t\left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G}\right) \quad (2.9.4)$$

which means that fiscal policy is financed by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$ in each country through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \quad (2.9.5)$$

while distributing equally among the two countries the cost of fiscal policy by varying jointly the tax rates in the following way:

$$\tau_t^{*s} - \tau^{*s} = \tau_t^s - \tau^s \tag{2.9.6}$$

$$\tau_t^{*w} - \tau^{*w} = \tau_t^w - \tau^w \tag{2.9.7}$$

where $\psi_{nx} \geq 0$ represents the responsiveness of government consumption to variations of the real net exports gap, $\rho_g \in [0, 1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while variables without subscript t represent their respective steady state level.

3 Calibration

The model is calibrated[8] following mainly Ferrero (2009), so we consider the top 5 Eurozone countries, which account for more than 80% of Eurozone GDP and we divide them into the periphery (namely, France, Netherlands, Italy and Spain), country F, and the core (namely Germany), country H. The size of country H is set according to the relative GDP size to $h = 0.4$, as Germany accounts for over 35% of Eurozone GDP.

As in Ferrero (2009) most of the parameters governing the economies of the two countries are set symmetrically, with the exception of the degree of price rigidity, which has been set such that in country H the average duration of a price is 4 quarters while in country F it is 5 quarters, to account for a greater price rigidity in the Eurozone periphery with respect to Germany. The gross markup $\frac{\varepsilon}{\varepsilon-1}$ has been set to 1.1, which implies a net markup of 10%, and the discount factor has been chosen to match a compounded annual interest rate of 2%. The parameters for monetary policy follow common values used in the literature, so we set the response of the interest rate to inflation to $\phi_\pi = 1.5$, according to the Taylor principle, and the interest rate smoothing parameter to $\rho_i = 0.8$. Table 1 collects all calibrated parameters and steady state stances.

In the calibration, we set $\eta > \frac{1}{\sigma}$ so that C_H and C_F are substitutes and hence the substitution effect of a price change dominates the income effect. In the opposite case $\left(\eta < \frac{1}{\sigma}\right)$ C_H and C_F are complements and the income effect of a price change dominates the substitution effect. This implies that fiscal policy and spillovers from one country to the other have very different effects based on the two calibrations. In our analysis we focus on the case in which C_H and C_F are substitutes because we believe it is more realistic, especially for advanced economies, and more in line with the recent literature (See Ferrero (2009) and Blanchard, Erceg and Lindé (2015) for instance), but we also consider the case in which they are complements, as a sensitivity

[8] The calibration is done on the steady state values described in Appendix A.2.

Table 1: Calibrated Parameters and Steady State Stances.

Parameter	Description	Country H	Country F
h	Relative size of domestic economy	0.4	0.6
β	Discount factor	0.995	0.995
ε	Elasticity of substitution of domestic goods	11	11
$\varepsilon/(\varepsilon-1)$	Gross Price Mark-Up	1.1	1.1
η	Elasticity of substitution foreign and domestic goods	[0.3, 4.5]	[0.3, 4.5]
σ	Inverse elasticity of intertemporal substitution	3	3
φ	Inverse Frisch Elasticity of labour supply	0.5	0.5
θ	Degree of price rigidity	3/4	4/5
α	Openness of domestic economy	0.52	0.361
$\alpha/(1-h)$	Relative openness of domestic economy	0.867	0.9025
$(1-\alpha)/h$	Home bias	1.2	1.065
ψ_y	Responsiveness of fiscal policy to output gap	0.045	0.001
ψ_{nx}	Responsiveness of fiscal policy to net exports gap	0.0697	0.0092
ϕ_π	Responsiveness of monetary policy to inflation	1.5	1.5
ρ_i	Interest Rate smoothing parameter	0.8	0.8
ρ_ξ	Persistence of preference shock	0.94	0.8
ρ_a	Persistence of technology shock	0.58	0.70
σ_ξ	Standard deviation of preference shock	0.0024	0.0086
σ_a	Standard deviation of technology shock	0.0087	0.0033
$corr_\xi$	Correlation of preference shock	0.625	0.625
$corr_a$	Correlation of technology shock	0.418	0.418
Steady State Ratios	**Description**	**Country H**	**Country F**
$(1+i)^4-1$	Annualized Interest Rate	2%	2%
τ^w	Tax rate on labour income	40.61%	27.94%
τ^s	Tax rate on firm sales	2.5%	19.5%
$\tau^w MC+\tau^s$	Tax revenues-to-GDP	38.49%	39.92%
G/Y	Government consumption-to-GDP	18.7%	21.9%
$\tilde{T}/Y$	Real transfers-to-GDP	18.58%	16.81%
$\widetilde{NX}/Y$	Net Exports-to-GDP	1.72%	-1.14%
C/Y	Consumption-to-GDP	79.58%	79.24%
$\alpha^* C^*/Y$	Exports-to-GDP	43.1%	27.47%

analysis for the effects of fiscal policy, as studied in Hjortsø (2012).

The calibration of the two countries mainly differs in the fiscal policy parameters. In particular, the government consumption-to-GDP ratios have been set respectively to 18.7% for country H (Germany) and 21.9% for country F, according to the average of the last 9 years (source ECB-SDW). The marginal tax rates on labour income have been set respectively to 40.61% for country H (Germany) and 27.94% for country F in accordance to the average in the last 9 years of the labour income tax wedges, excluding social security contributions made by the employer, for the median individual, as reported in OECD (2015). The marginal tax rate on firm sales has been set to 19.5% for country F according to the average VAT in the last 9 years for France, Italy, Spain and The Netherlands as reported in Eurostat, European-Commission et al. (2015), while it has been calibrated for country H to match the average ratio of net exports-to-GDP of 1.73% observed over the past 9 years for Germany[9]. Although the observed VAT rate for Germany is 19%, we set its marginal tax rate on firm sales to 2.5%, as if there were a production incentive, to correct for the fact that country H should have a greater productivity compared to country F, as Germany has a greater productivity than the periphery countries. This calibration implies a steady state tax revenue-to-GDP ratio of respectively 38.49% for country H and 39.92% for country F, clearly in line with the data observed over the past decades for Germany (38.72%) and for France, Italy, Spain and The Netherlands (39.15%). Finally, the annualized steady state value of government debt-to-GDP in both countries is set to roughly 60% as stated in the Maastricht Treaty.

Since the two countries' fiscal policy ratios have been calibrated according to the data, the transfers-to-GDP ratios have been set such that the government deficit is zero in steady state, which for country H reads:

$$\frac{\tilde{T}}{Y} = (\tau^s + \tau^w MC) - \frac{G}{Y} - \left(\frac{1}{\beta} - 1\right)\frac{\tilde{B}^G}{Y} \tag{3.0.1}$$

Henceforth, the overall calibration of the fiscal sector implies a steady state ratio of transfers-to-GDP of respectively 18.58% for country H and 16.81% for country F, and a steady state ratio of current expenditure-to-GDP of respectively 37.28% for country H and 38.71% for country F. This calibration is broadly in line with the observed data over the last 10 years for the subsidies-to-GDP ratio (26.85% for Germany and 24.69% for the other countries) and the current expenditure (less interest)-to-GDP ratio (35.54% for Germany and 36.85% for the other countries).

The parameters of openness have been set to match an export-to-GDP ratio $\left(\frac{\alpha^* C^*}{Y}\right)$ of roughly 43% for country H[10] (Germany) taken from the aggregate demand equation, while for country F the parameter

[9]The average current account to GDP ratio observed over the past 9 years for Germany is roughly 6.36%. However, we adjust the data for the overall trade weight with France, Italy, Spain and The Netherlands (26%).

[10]The value recovered from the data as the average of the last 9 years is 43.5%.

of openness is recovered by equating per-capita consumption across countries, which yields the following equation:

$$\alpha^* = \frac{h}{1-h}\left[\alpha + \frac{\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\right)\left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})}\right)^{\frac{1}{\varphi}} - 1}{1+\frac{h}{1-h}\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\right)\left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})}\right)^{\frac{1}{\varphi}}}\right] \tag{3.0.2}$$

Consequently, relative home biases are given by $\frac{1-\alpha}{h} = 1.2$ and $\frac{1-\alpha^*}{1-h} = 1.065$. Since both home biases are larger than one it means that the share of consumption of domestic goods is higher than the share of production of domestic goods, which means exactly that household consumption is biased domestically.

Regarding the dynamic parametrization of the shocks, all exogenous shocks are assumed to follow a $VAR(1)$ process that generally allows for both direct spillovers and second order correlation of the innovations. However, the structure has been restricted for both the technology shocks and the preference shocks to exclude direct spillovers.

With the exception of the preference shocks, whose dynamics have been calibrated following Kollmann et al. (2014), the parameters characterizing the dynamics of the technology shocks have been estimated employing the time series for Germany, France, Italy and Spain of labour productivity per hours worked. All the series are chain-linked volumes re-based in 2010, seasonally adjusted and filtered by means of a Hodrick-Prescott filter. The sample considered spans at quarterly frequency from 2002 Q1 to 2015 Q3. Finally, despite a large debate on the high correlation between preference shocks in the Eurozone, there is no proper reference in the literature for its calibration. We decide to set this parameter according to the observed business cycle correlation (which is roughly 0.5) and we pick the value that maximizes the simulated correlation between output in the two countries (which is roughly 0.42)[11].

4 Welfare Analysis and Optimal Fiscal Policy Parameters

The selection of the optimal fiscal policy parameters follows from the analysis of the fiscal policy rules used in our model. In the search for the optimal fiscal policy parameters, we limit the analysis to an expenditure rule that sets government consumption to target either the output gap or the net exports gap, as in our model equations. Specifically, these fiscal policy parameters have been selected to maximize the unconditional expectation of lifetime utility of the total population of households[12] under the condition that they induce a

[11] The simulated values of the correlation of business cycles in our model, given our calibration, are always lower than the observed correlation. Therefore, we decide to select the correlation of preference shocks that maximizes the correlation of business cycles.

[12] Even if in the Pure Currency Union scenario the fiscal decisions are taken independently, we consider the results of the joint maximization of average aggregate welfare because it is in line with the results of a dynamic game between the two countries.

locally unique rational expectations equilibrium[13]. Therefore the fiscal policy rules in our model are judged optimal in their class of rules, because the fiscal policy parameters are chosen to yield the highest average level of welfare to the representative household compared to all other fiscal policy parameters.

As a measure of welfare we consider the weighted average of the second order approximation of the utility of households in each country, given by:

$$\widetilde{W}_t = hW_t + (1-h)W_t^* \tag{4.0.1}$$

where welfare for country H is given by:

$$W_t = \xi_t \left(\frac{\left(\frac{C_t}{h}\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(\frac{Y_t d_t}{A_t h}\right)^{1+\varphi}}{1+\varphi} \right) + \beta W_{t+1} \tag{4.0.2}$$

and given by an analogous equation for country F.

Although we select the fiscal policy parameters based on the unconditional expectation of lifetime utility, to compare welfare attained under alternative fiscal policy scenarios we prefer to rely on the expectation of lifetime utility conditional on the initial state being the non-stochastic steady state. In this way, the welfare ranking of alternative policies will depend on the assumed value and distribution of the initial state vector (x_0). This measure accounts for the transitional dynamics leading back to the stochastic steady state and, as the deterministic steady state is the same across all the scenarios considered, we ensure that the economy begins from the same initial point under all possible policies.

4.1 Welfare Costs based on Consumption Equivalent Variations

Following Schmitt-Grohé and Uribe (2007), we compute the welfare cost of a particular fiscal policy scenario relative to our benchmark scenario: the Pure Currency Union scenario with exogenous government consumption. We denote the benchmark policy scenario with b, the alternative scenarios with a, and the steady state scenario with 0, and we consider the welfare cost λ as the percentage decrease in the benchmark scenario's expected consumption that leaves the representative household as well off as in the alternative

[13]Following Schmitt-Grohé and Uribe (2007), we discretize the policy space by means of a grid search, because welfare is a non monotonic function of the fiscal policy parameters and has several local maxima. We consider 100 different values for each target variable (i.e. either the output gap or the net exports gap) and limit the parameter space to lie between 0 and 0.1 or between -0.1 and 0 based on the expected sign of the parameter, because a larger parameter space would imply a non stationary equilibrium given by the distortionary effect of taxation overcoming the stabilizing effect of government spending.

scenario. Therefore λ can be recovered from the following identity:

$$E\{W_a\} = \frac{\xi_b}{(1-\beta)}\left(\frac{\left(\frac{(1+\lambda)C_b}{h}\right)^{1-\sigma}-1}{1-\sigma} - \frac{\left(\frac{Y_b d_b}{A_b h}\right)^{1+\varphi}}{1+\varphi}\right) =$$

$$(E\{W_b\} - W_0)(1+\lambda)^{(1-\sigma)} + \frac{(1+\lambda)^{(1-\sigma)}-1}{(1-\sigma)(1-\beta)} + W_0 \quad (4.1.1)$$

The welfare cost is then equal to:

$$\lambda = 1 - \left[\frac{(1-\sigma)(E\{W_a\} - W_0) + (1-\beta)^{-1}}{(1-\sigma)(E\{W_b\} - W_0) + (1-\beta)^{-1}}\right]^{\frac{1}{1-\sigma}} \quad (4.1.2)$$

Note that in the equation above λ is a function of both the initial conditions (x_0) and the expected variance (σ_0), because it is a function of the conditional expectations of welfare, which in turn depend on x_0 and σ_0. To compute the value of λ we consider its Taylor expansion around the point $x = x_0$ and $\sigma_0 = 0$. Since we choose the initial state to be the deterministic steady state, we need to consider a second-order approximation of λ because only the second derivatives of welfare with respect to σ are non-zero. Indeed, since the steady state is the same across all scenarios, λ vanishes around the point (x_0, σ_0) and the first derivatives with respect to σ are null. Totally differentiating twice the welfare gain λ and evaluating the results at (x_0, σ_0) yields:

$$\lambda \approx \left(\frac{\partial^2 W_a}{\partial \sigma^2} - \frac{\partial^2 W_b}{\partial \sigma^2}\right)(1-\beta) \quad (4.1.3)$$

The optimal fiscal policy parameters and the welfare costs based on Consumption Equivalent Variations are reported in Table 2. The optimal coefficients have been selected respectively under the Pure Currency Union scenario for the response of government consumption to the output gap and under the Coordinated Currency Union scenario for the response of government consumption to the net exports gap.

Table 2: Optimal Fiscal Policy Parameters and Welfare Costs based on CEV

Policy Scenarios	Optimal Parameters*		Conditional Welfare Costs		
	ψ	ψ^*	Country H	Country F	Average
PCU (exogenous)	0	0	0%	0%	0%
PCU	0.067	0.061	0.32%	0.27%	0.29%
CCU	0.043	0.014	0.01%	0.28%	0.17%
FFU	0.043	0.014	-0.02%	0.32%	0.19%
FFU (exogenous)	0	0	0.48%	0.33%	0.39%

*The optimal parameters have been selected by maximizing the unconditional expectation of lifetime utility.

Our analysis shows that using a targeting rule, whichever the target, rather than having exogenous government consumption, is always welfare detrimental for a country because, although rules stabilize a variable, they require a tax adjustment which produces large distortions. This is true if the two countries are independent with separate budget constraints, while if they form a fiscal union with a consolidated budget constraint using a targeting rule is welfare improving because the consolidation of budget constraints and the joint movement of the tax rates provide greater welfare costs which imply greater gains from stabilization (especially for country H). From Table 2 we can see that the average welfare cost is lowest in the Coordinated Currency Union scenario compared to other scenarios (with targeting), because stabilizing net exports is found to be welfare improving compared to stabilizing output. Second place for average welfare is held by the Full Fiscal Union scenario, which has a smaller welfare cost compared to the the Pure Currency Union scenario, mainly because the latter compared to the former reduces a lot welfare in country H. The response of government consumption to domestic output induces large fluctuations in distortionary taxes in both countries, but if government consumption targets net exports, the implied tax volatility is mitigated by reduced international spillovers and reduced volatility in the terms of trade, which benefit country H. In the Full Fiscal Union scenario the distortionary effects created by the consolidation of budget constraints are similar to those under the Pure Currency Union scenario with targeting. However, the welfare costs induced by tax fluctuations decrease significantly when net exports are stabilized, as they also stabilize the terms of trade.

The lowest welfare cost for country H is in the Full Fiscal Union scenario (with targeting), because it is the country with positive net exports and thus has more to gain in stabilizing the net exports gap. Also, since country H has a lower degree of price rigidity, after a shock prices move more than in country F, so that in country H the direct effect on output (income effect) and the indirect effect through the terms of trade (substitution effect) move in the same direction, bringing the economy further away from the initial equilibrium. Thus, stabilizing net exports yields lower welfare costs because it counteracts the substitution effect, reducing the negative effects of the shock. Note that, although small, there is a welfare gain for country H in the Full Fiscal Union scenario (with targeting) compared to the Coordinated Currency Union scenario, given by the distributional effects of the consolidation of budget constraints, that puts more burden of financing on the country with higher output (country F).

The lowest welfare cost for country F is instead in the Pure Currency Union scenario, because it is the country with negative net exports and thus has more to gain in stabilizing the output gap. Also, since country F has a higher degree of price rigidity, after a shock prices move less than in country H, so that in country F the income effect and the substitution effect move in opposite directions. Stabilizing output instead of net exports yields smaller welfare costs because it allows country F to partly offset the higher

degree of price rigidity by letting the terms of trade and thus net exports fluctuate freely. Given that taxes move jointly in the Full Fiscal Union scenario, after a shock in country H, taxes in country F must fluctuate much more on impact and, given the higher degree of price rigidity compared to country H, the distortionary effect is very persistent. On the other hand, after a shock in country F the movements in taxes are smaller on impact and so the effect is less persistent. For these reasons, the Full Fiscal Union scenario produces the highest welfare cost for country F and the smallest for country H. Based on this welfare criterion, it seems that Germany has more to gain from a fiscal union than the rest of the Eurozone.

4.2 Welfare Gains based on an ad hoc Loss Function

Blanchard, Erceg and Lindé (2015) argues that utility-based welfare measures probably underestimate the benefits of reducing the output gap in economies facing a high resource slack (negative net exports), as in the Eurozone periphery. Explicitly, a utility-based welfare measure shows less benefits from fiscal expansions than a simple ad hoc welfare measure, because net exports play a substantial role in reducing the periphery's output gap and the increase in consumption in the periphery is delayed so that it has very small welfare effects.

We decide to compare the policy scenarios also based on an ad hoc loss function for the reasons stated above, as in Blanchard, Erceg and Lindé (2015). Since fiscal policy has a stabilizing function, it mimics the behavior of monetary policy, and together they reduce both the inflation gap and the output gap. Furthermore, there are gains in terms of consumption and unemployment related to closing the output gap that are underestimated by utility-based measures. Hence, the gains from fiscal spillovers between the core and the periphery lie between the ones based on consumption equivalent variations and the ones based on an ad hoc loss function.

Using a standard quadratic loss function, the policymakers are assumed to care only about minimizing the square of the output gap and of the inflation gap in both regions. Each region's loss function is, hence, simply the sum of the square of the inflation gap and the square of the output gap, with weights 3 and 1 respectively. The overall loss function is the weighted average of each region's loss function, given by:

$$Loss = \sum_{j=0}^{\infty} \beta^j \left\{ h \left[(\hat{\pi}_{t+j})^2 + \frac{1}{3} (\hat{Y}_{t+j})^2 \right] + (1-h) \left[(\hat{\pi}^*_{t+j})^2 + \frac{1}{3} (\hat{Y}^*_{t+j})^2 \right] \right\} \tag{4.2.1}$$

where variables with a hat ($\hat{}$) indicate their log-deviation from steady state.

From Table 3 we can see that there are welfare gains in the Coordinated Currency Union and in the Full Fiscal Union scenarios with respect to the baseline scenario of a Pure Currency Union, because targeting the net exports gap reduces the overall inflation gap and consolidating budget constraints reduces the output

gap, providing overall stabilization for both countries. At the same time targeting the output gap (second line in table 3) is welfare reducing, compared to stochastic government consumption (first line in table 3), which means that output is stabilized more by targeting net exports or nothing. The average welfare gain is greater in the Full Fiscal Union scenario compared to other scenarios. Second place for average welfare is held by the Coordinated Currency Union scenario, which has welfare gains compared to the Pure Currency Union scenario, mainly because targeting the net exports gap reduces international spillovers by stabilizing overall output and inflation, with little difference in welfare gains compared to the Full Fiscal Union scenario.

Table 3: Welfare Gains based on an ad hoc Loss Function

Policy Scenarios	Losses			Welfare Gains*		
	Country H	Country F	Average	Country H	Country F	Average
PCU (exogenous)	0.2207	0.1832	0.1982	0	0	0
PCU	8.6143	7.3293	7.8433	-3803%	-3900%	-3857%
CCU	0.0085	0.0046	0.0062	96.16%	97.46%	96.88%
FFU	0.0054	0.0028	0.0038	97.57%	98.47%	98.07%
FFU (exogenous)	0.0043	0.0026	0.0033	98.03%	98.56%	98.32%

*Welfare Gains are computed as $\frac{Loss_b - Loss_a}{Loss_b}$, with $Loss_b$ the loss in the PCU with $\psi = \psi^* = 0$.

We can see from Table 3 that the welfare gains for the two countries, both individually and on average, are increasing in the degree of coordination. The fact that both countries incur in big welfare losses in the Pure Currency Union scenario (second line in table 3), while incurring in welfare gains in the other scenarios, shows the big welfare gains from either targeting the net exports gap compared to the output gap or consolidating budget constraints. As a matter of fact, adding one dimension to the other makes almost no difference in welfare terms, as the gains take place by *either* targeting net exports *or* consolidating budget constraints. What is quite surprising is that, according to this welfare measure, only consolidating budget constraints (fifth line in table 3) yields welfare gains similar and a little greater to those achieved by only targeting the net exports gap (third line in table 3). This is because a consolidated budget constraint stabilizes the inflation gap (by moving tax rates jointly) and the output gap (by moving tax rates less) on its own, in a similar manner to what targeting the net exports gap does.

5 Numerical Simulations

We simulate the model numerically using Dynare[14], which takes a second-order approximation of the model, following Schmitt-Grohé and Uribe (2004), around its symmetric non-stochastic steady state with zero

[14]All the equilibrium conditions of the model used for the simulations are shown in Appendix A.1.

inflation and constant government debt. We compare the impulse response functions of the main variables to negative supply and demand shocks of one standard deviation, under a range of fiscal policy specifications, to study the stabilization properties of different coordination strategies and financing schemes.

In the following graphs we analyze the impulse responses to a negative technology shock in country H and the impulse responses to a negative preference shock in country F. These two shocks account well for the dynamics in the Eurozone, because a supply shock is more relevant in a country like Germany (country H), which is a main producer and exporter of goods and services, while a demand shock is more relevant for periphery countries (country F), which are mainly consumers and importers.

5.1 Fiscal Policy Coordination

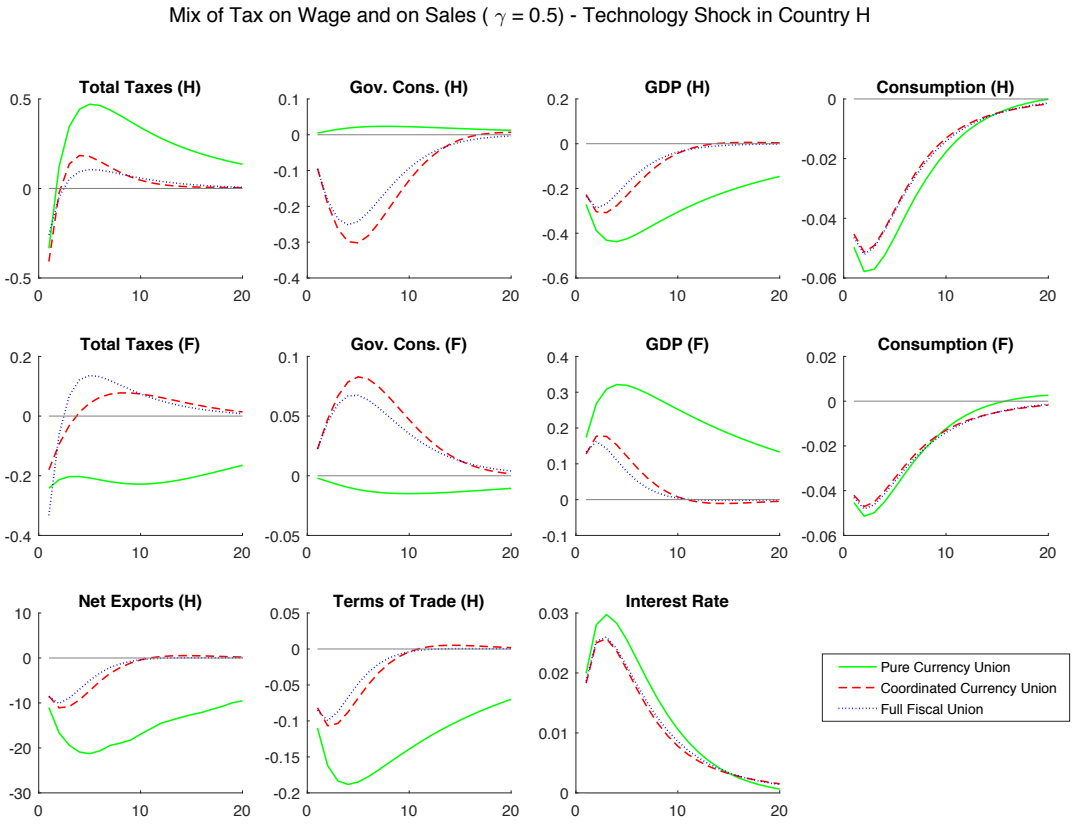
In the following graphs we compare the three different degrees of fiscal policy coordination - Pure Currency Union, Coordinated Currency Union, and Full Fiscal Union - after a negative technology shock in country H and after a negative preference shock in country F[15]. The impulse responses are shown in Figure 2 and Figure 3, respectively.

After a negative technology shock in country H, marginal costs increase, bringing to an increase in prices and a decrease in output. The decrease in output brings to an increase in taxes to balance the government budget, which pushes prices and thus domestic inflation to rise, reinforcing the effect on prices of the increase in marginal costs. The consequent monetary policy tightening drives lower consumption in both countries, due to the assumption of complete markets. Since prices in country H are more flexible than those in country F, the terms of trade fall, inducing a deterioration in net exports for country H. This, in turn, amplifies the recession in country H and determines an expansion in country F, amplified by the decrease in taxes or the increase in government consumption. A negative preference shock in country F, instead, decreases consumption and thus prices and inflation in country F, inducing higher labour supply and thus output, which induces lower taxes to balance the budget and thus lower prices and inflation in country F, reinforcing the effect on prices of the drop in consumption. The reduction in overall inflation pushes the common central bank to lower the interest rate, inducing an increase in private consumption in country H. As observed for the technology shock, the terms of trade drop, in this case also due to the opposite dynamics of consumption, inducing net exports to fall, thus amplifying the recession in country H and the expansion in country F.

Looking at Figures 2 and 3, we can see that the response of the national fiscal authorities varies according to the fiscal policy scenario. In the Pure Currency Union scenario (solid green line), countercyclical fiscal policy implies an increase (decrease) in government consumption given a decrease (increase) in domestic

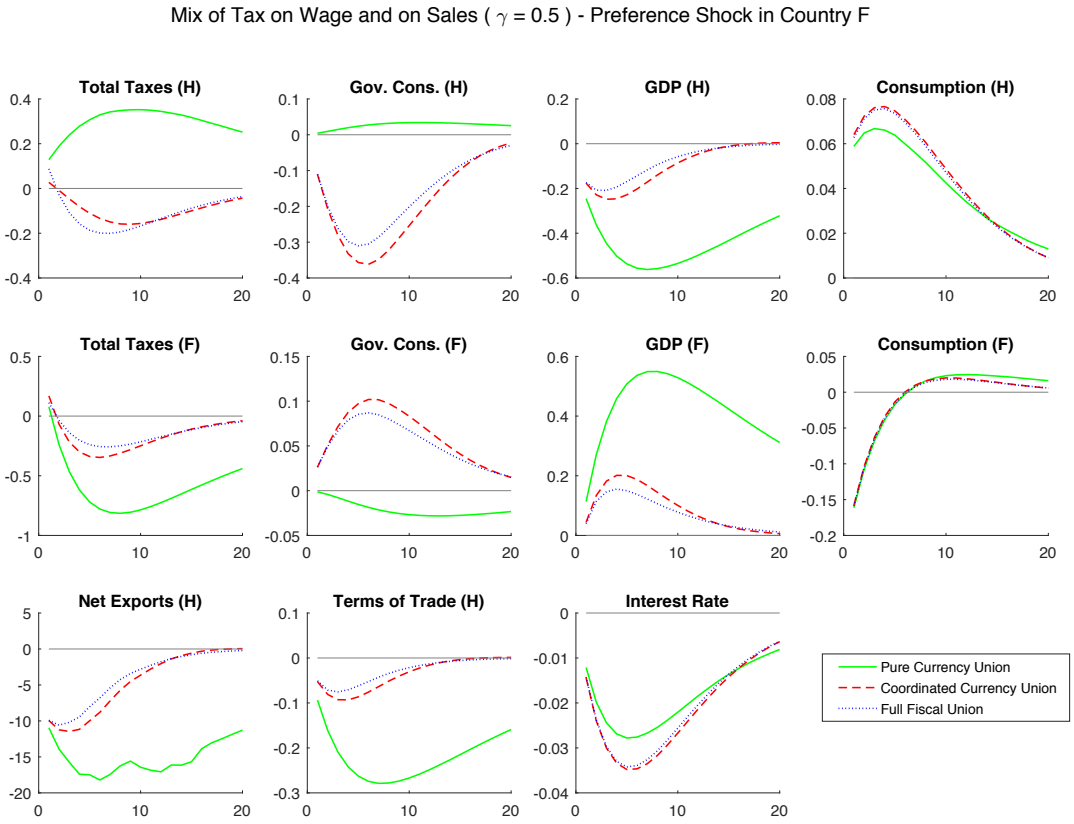
[15]The financing scheme for these simulations is given by a balanced mix of the two tax rates, corresponding to the case $\gamma = \gamma^* = 0.5$ analyzed in the Section 5.2. Even if we show that the amplification of the shocks is increasing in *gamma*, we prefer to use balanced financing ($\gamma = 0.5$) for all other simulations, as the tax mix does not affect qualitatively the dynamics.

Figure 2: Fiscal Policy Coordination - Technology Shock in Country H



output, which are then balanced by varying the tax rates to balance the government budgets. This determines in some cases more amplified dynamics of output, since the stabilizing effect of government consumption on output is offset by the movements in distortionary taxes, which affect consumption and prices a lot. On the other hand, by targeting net exports in the other two scenarios, government consumption decreases in country H and increases in country F. In this case the tax dynamics do not follow closely government consumption, but still follow the opposite dynamics of GDP (the tax base) to balance the government budgets. Moreover, when government consumption targets net exports rather than output, the terms of trade are less volatile, so that international spillovers (net exports) are reduced and the economy is more stable. Specifically, net exports fluctuate highly after a shock, due to the re-balancing of household consumption baskets, following movements in the terms of trade. By stabilizing net exports the terms of trade are consequently more stable, reducing the international substitution effect. As a result, the dynamics are much more amplified under the Pure Currency Union scenario and much less amplified under the Coordinated Currency Union scenario

Figure 3: Fiscal Policy Coordination - Preference Shock in Country F

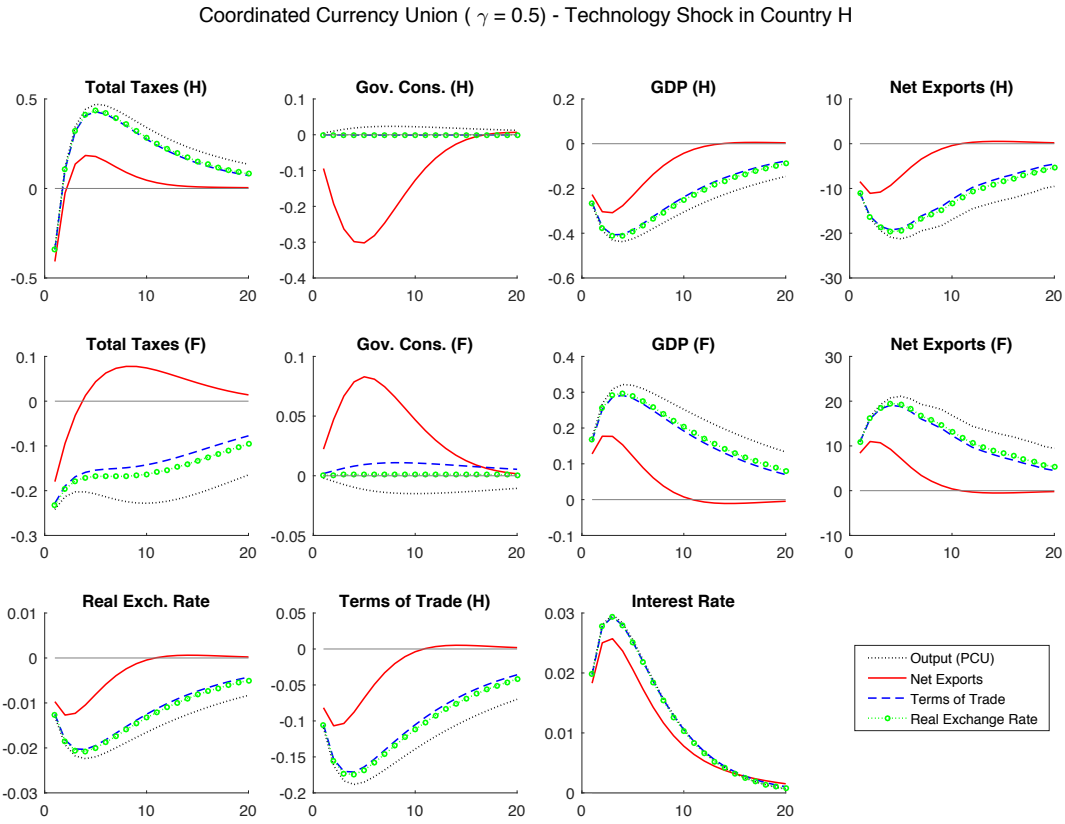


(dashed red line).

The Full Fiscal Union scenario (dotted blue line) presents dynamics which are very close to those in the Coordinated Currency Union scenario, because in both cases government consumption targets net exports. As a consequence, we provide evidence of a significant gain in terms of stabilization when the government targets net exports, while if we also consolidate budget constraints we obtain very small improvements in terms of stabilization (the dashed red line and the dotted blue line follow very close paths). However, the joint movement in the tax rates makes the terms of trade more stable, reducing international spillovers and bringing government consumption to react less in the Full Fiscal Union scenario compared to the Coordinated Currency Union scenario, which brings to more stable dynamics in output in both countries.

As a robustness check with respect to the common international target for fiscal policy coordination, we simulate the model using alternative common international targets, like the real exchange rate or the terms

Figure 4: Targets For Coordination - Technology Shock in Country H



of trade[16]. In Figure 4 we compare the dynamics after a negative technology shock in country H of four different targets for government consumption: domestic output in the Pure Currency Union scenario (dotted gray line), net exports (solid red line), the terms of trade (dashed blue line) and the real exchange rate (big dotted green line) under the Coordinated Currency Union scenario.

After a negative technology shock in country H, the terms of trade and the real exchange rate fall, bringing consequently to a fall in net exports and inducing country H to reduce government consumption, while country F increases it, with all fiscal policy targets except for output. Since net exports fall in country H, GDP falls in country H and rises in country F, bringing taxes to rise in country H and fall in country F to balance the government budget. The overall inflationary pressure determines a more aggressive monetary policy tightening compared to the case in which the common international target is net exports. The higher

[16]The coefficients for the response of government consumption to either the real exchange rate or the terms of trade have been selected to maximize the unconditional expectation of the lifetime utility of households under the condition that they induce a locally unique rational expectations equilibrium.

interest rate amplifies consumption and thus output dynamics in both countries, making the stabilization of international variables less effective. Furthermore, total taxes in country F follow the opposite path of GDP with all targets except for net exports. This reversal of the dynamics of total taxes with target net exports (solid red line) with respect to other targets is given by the much smaller increase in the tax base (GDP) and much greater increase in the response of government consumption, which brings taxes to increase rather than decrease, stabilizing relative prices and thus net exports more than with other targets.

This analysis confirms that only reducing international demand imbalances it is possible to offset the international substitution effect and to create positive spillovers between fiscal and monetary policy that lead to more stable dynamics in the economy.

5.2 Financing Fiscal Policy

Here we compare different financing schemes for fiscal policy, varying the percentage financed by the tax rate on labour income with respect to the tax rate on firm sales. More in detail, we simulate the model under three combinations of τ^s and τ^w:

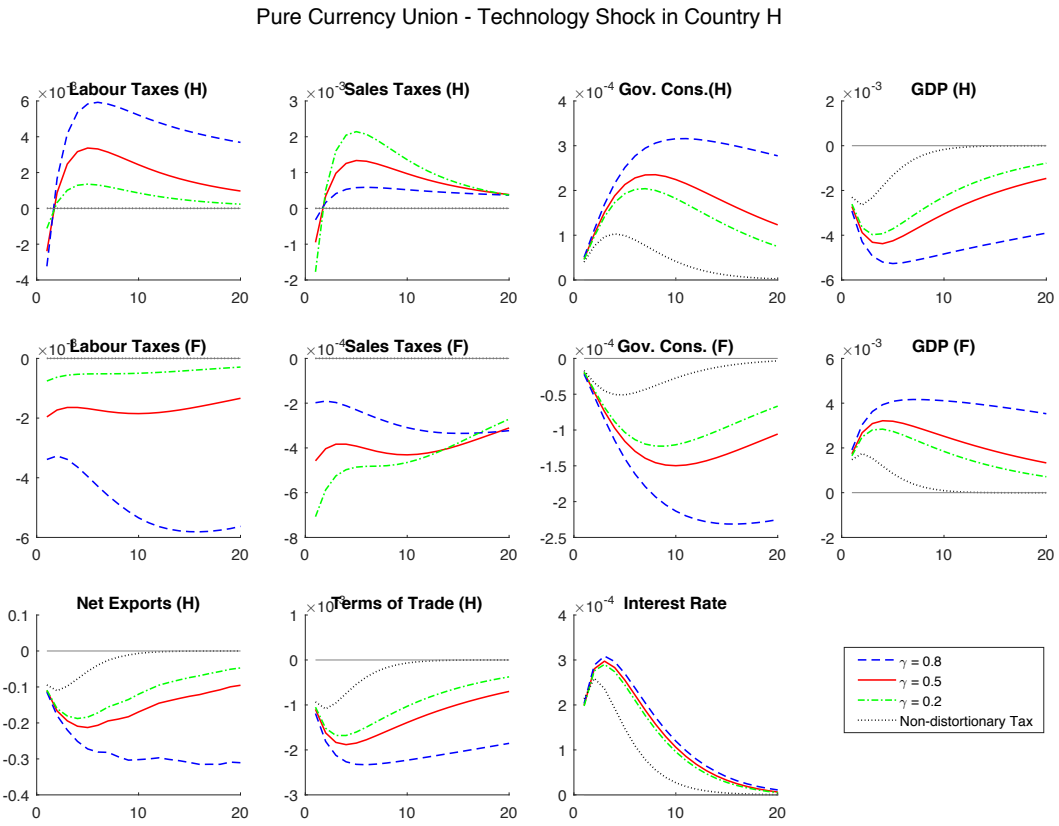
- $\gamma = 0.2$, financed roughly 20% by varying the tax rate on labour income and 80% by varying the tax rate on firm sales.
- $\gamma = 0.5$, financed roughly by varying equally the two tax rates. This can be considered as the baseline financing scheme, followed in all other simulations.
- $\gamma = 0.8$, financed roughly 80% by varying the tax rate on labour income and 20% by varying the tax rate on firm sales.

We also compare the outcomes of financing fiscal policy with lump-sum taxes, which do not produce distortions in the economy. Figures 5 and 6 show the impulse responses with different financing schemes to a negative technology shock in country H in the Pure Currency Union scenario and in the Full Fiscal Union scenario, respectively[17].

In the Pure Currency Union scenario (Figure 5), when distortionary taxation is used by the governments, the dynamics are much more volatile than in the case in which lump-sum taxes finance government expenditure. As an example, if we compare the financing scheme in which the burden is shared equally by the two tax rates (solid red line) with the case in which non-distortionary taxation is used (dotted gray line), we can observe that both interest rate and output are much more stabilized with the latter financing scheme. Furthermore, the amplification of the shocks is increasing exponentially in γ, with the most amplified dynamics

[17]The results are similar after a preference shock in country F, so we don't show the impulse responses.

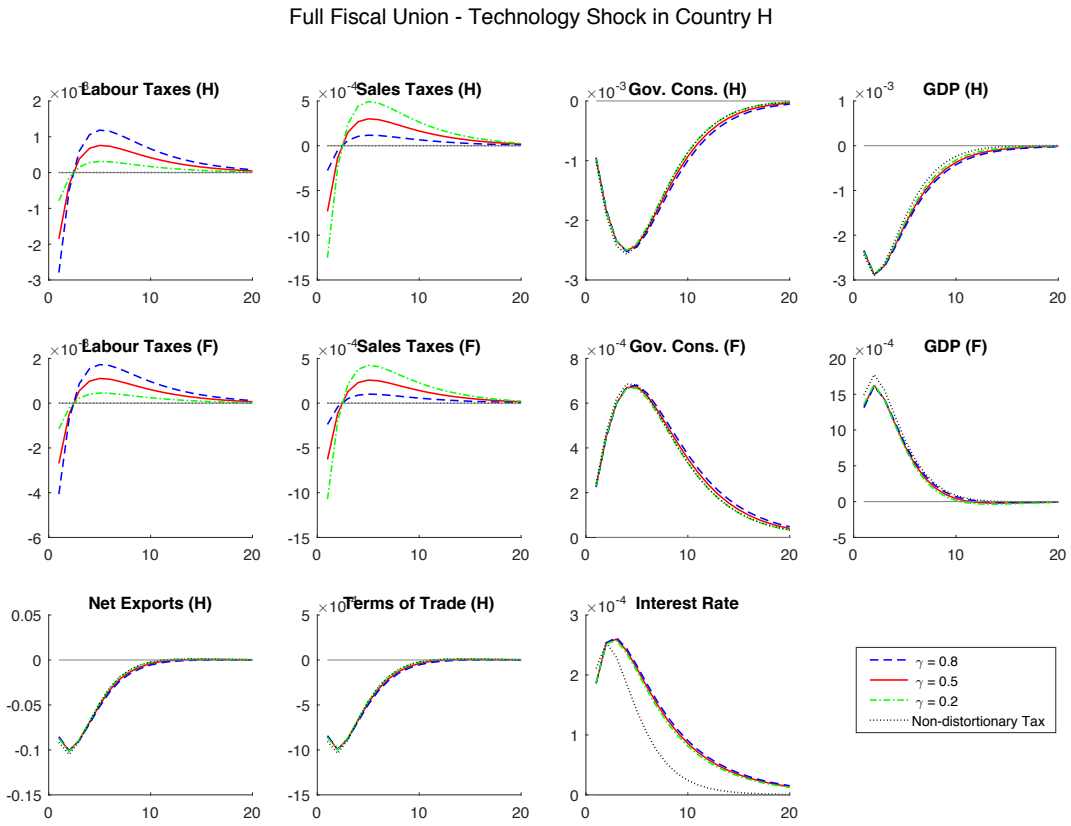
Figure 5: Fiscal Financing - Pure Currency Union - Technology Shock in Country H



given by the massive use of the tax rate on labour income ($\gamma = 0.8$, dashed blue line) to finance fiscal policy. When governments use distortionary taxation the most stable dynamics are given by varying mainly the tax rate on firm sales ($\gamma = 0.2$, dashed-dotted green line), while varying equally the two tax rates ($\gamma = 0.5$, solid red line) creates a little more distortion compared to $\gamma = 0.2$ and much less distortion compared to $\gamma = 0.8$. Therefore these results point out to the fact that taxes on labour income are much more distortionary than taxes on firm sales.

Our analysis implies that fiscal policy has a greater stabilization role when it is mainly financed by taxes on firm sales. In fact, the tax rate on labour income affects very much the equilibrium level of output and prices because of its direct impact on labour supply and marginal costs, while the tax rate on firm sales affects primarily prices and inflation, with secondary effects on output. Furthermore, the income effect of a variation in taxes on firm sales is smaller than that of a variation in taxes on labour income, because in the former case households can counteract partially the effect by rebalancing their consumption baskets with

Figure 6: Fiscal Financing - Full Fiscal Union - Technology Shock in Country H

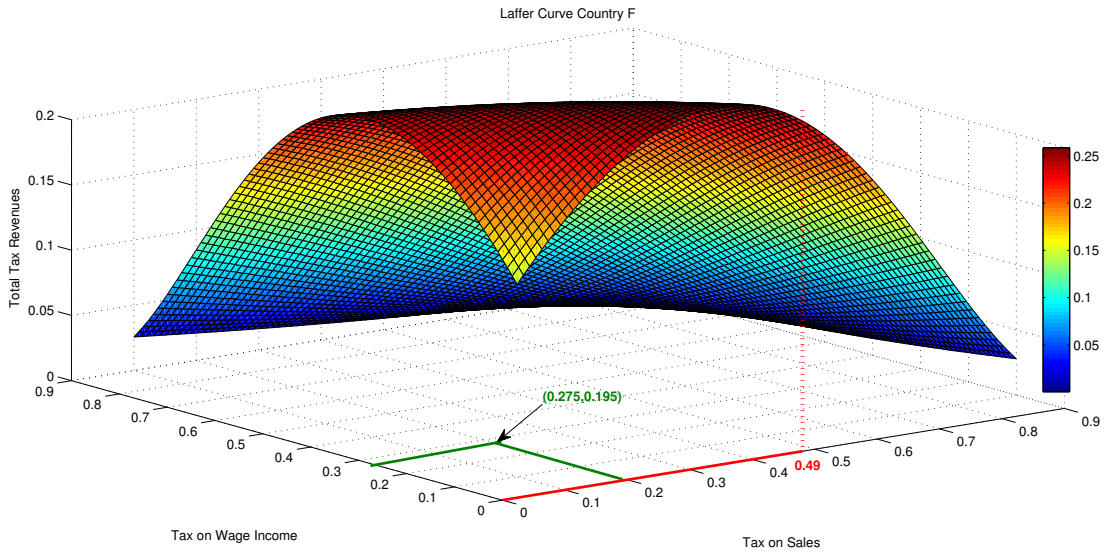


more or less imported goods. Notice that the volatility of output and of the terms of trade increase in γ, while what makes a difference for inflation and, in turn, for the interest rate, is the presence of distortionary rather than non-distortionary taxation.

As a robustness check, we assess how much the higher distortion given by taxing labour income is due to nominal price rigidities. We simulate the model assuming that prices are fully flexible[18] in both countries and we still observe that the amplification of the shocks is increasing in γ. With respect to the case of sticky prices, lump-sum taxes appear to be much more stabilizing than distortionary taxes, when prices are flexible. Therefore, we can infer that nominal price rigidities reduce the distortionary effects of taxation because prices react less after a shock and are less sensitive to adjustments in marginal costs. Although we might expect taxes on labour income to be less distortionary with sticky wages, previous literature, such as Forni, Gerali and Pisani (2010), generally finds that taxes on labour income are more distortionary than

[18] We do not show the impulse responses, although they are available on request.

Figure 7: Laffer Curve for Country F



taxes on firm sales, also with wage rigidity.

Figure 6 shows that, in the Full Fiscal Union scenario, the amplification induced by distortionary taxation holds only for taxes and thus prices and the interest rate, while output and the terms of trade are as stable as in the case of non-distortionary taxation. The greater stabilization in output is mainly due to the larger fiscal capacity available to country H after the shock, that allows taxes to move less. This is also because of the joint movement in the tax rates, which brings country F to increase rather than decrease taxes and government consumption, while country H decreases instead of increasing government consumption, with respect to the Pure Currency Union scenario. Moving government consumption and output in the same direction in both countries implies a smaller adjustment in taxation, stabilizing relative prices and international spillovers more.

Figure 7 complements our analysis by showing how the tax revenue is sensitive to the tax mix in the long-run. Specifically, the figure plots the steady state value for the total tax revenues given by varying the two steady state tax rates for country F. The tax rates on labour income and firm sales that maximize overall tax revenues are shown in red, while the calibrated tax rates on labour income and firm sales are shown in green.

We can see from the graph that the revenue maximizing value for the tax rate on labour income is zero because it is highly distortionary, as in the short run. This implies that the distortion created by the increase in the labour income tax (which reduces the tax base and thus revenues) dominates the increase in the tax

revenues. On the other hand, the revenue maximizing value for the tax rate on firm sales is between 40% and 50%, which implies it is much less distortionary compared to the tax rate on labour income, so the burden of adjustment should be mainly borne by the tax rate on firm sales. The results for country H are in line with those for country F, so we do not show its Laffer Curve.

6 The Case for International Goods as Complements

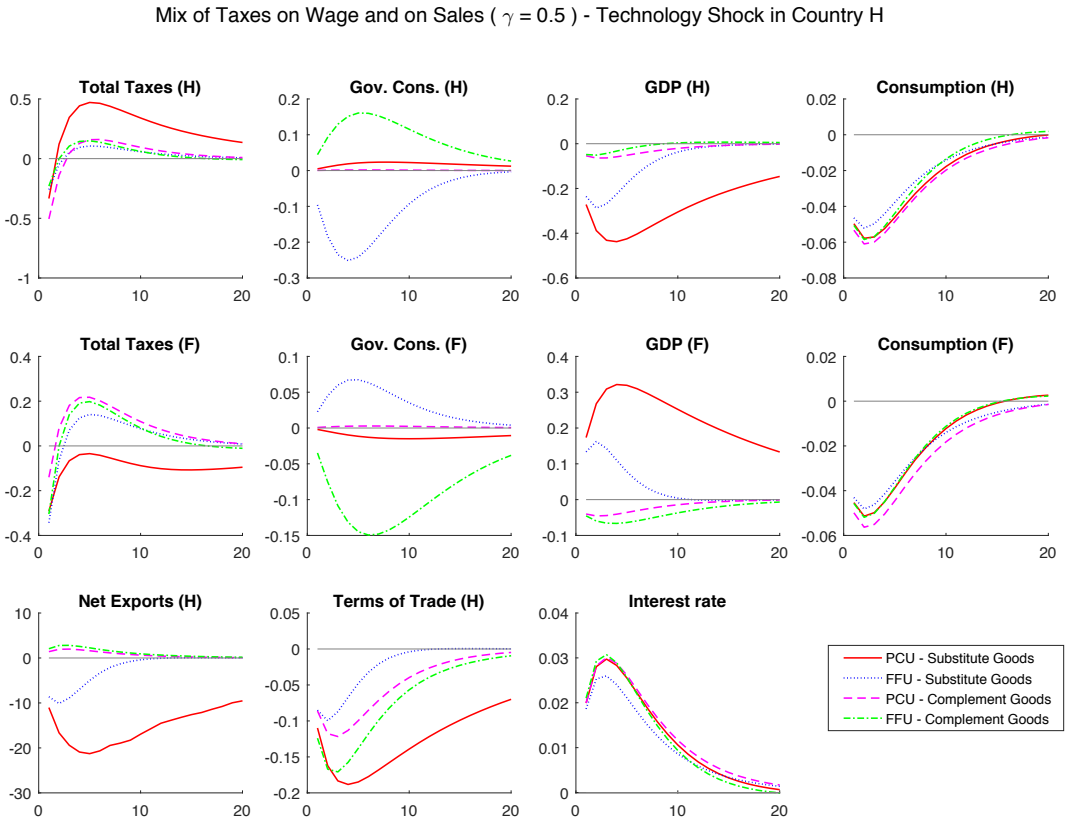
In the discussion above we highlighted the role played by international demand gaps in shaping the transmission of either demand or supply shocks, when the substitution effect of a price change dominates the income effect, because international goods are substitutes. However, Hjortsø (2012) shows that this result holds only with a high international trade elasticity ($\eta > \frac{1}{\sigma}$), while the opposite holds with a low international trade elasticity, because international goods are complements ($\eta < \frac{1}{\sigma}$). More importantly, this affects the ability of fiscal policy to reduce international demand imbalances, given by fluctuations in the terms of trade and in net exports. For these reasons, we assess how much the previous results are sensitive to the alternative assumption of international goods as complements, instead of substitutes, by changing the calibration of η.

Figure 8 shows the impulse responses to a negative technology shock in country H in the Pure Currency Union scenario and in the Full Fiscal Union one, comparing the two cases of international goods as substitutes and complements. After a technology shock in country H, PPI inflation becomes relatively higher in country H compared to country F. As a result the terms of trade deteriorate and, when international goods are substitutes, the substitution effect of the terms of trade change dominates the income effect, lowering net exports and output for country H. When domestic and foreign goods are instead complements, the income effect of the terms of trade change dominates the substitution effect, spurring net exports and reducing the recession in country H.

Looking at Figure 8 we can see that in the Full Fiscal Union scenario the overall volatility of the economy is substantially lower, in particular that of output and net exports, *only* when international goods are substitutes. When international goods are complements, instead, there is little difference between the two scenarios. Moreover, most variables, especially net exports and GDP, follow much more stable paths when international goods are complements. This finding is consistent with Hjortsø (2012), which highlights the fact that the cross-country insurance role of fiscal policy is more effective in closing demand imbalances when the internationally traded goods are complements instead of substitutes. More in detail, Figure 8 shows that, although with complement goods the dynamics are less amplified altogether, the Full Fiscal Union scenario produces quite more volatility for the terms of trade in country H and GDP in country F.

Figure 9 provides evidence that our considerations above, regarding the lower volatility of net exports and

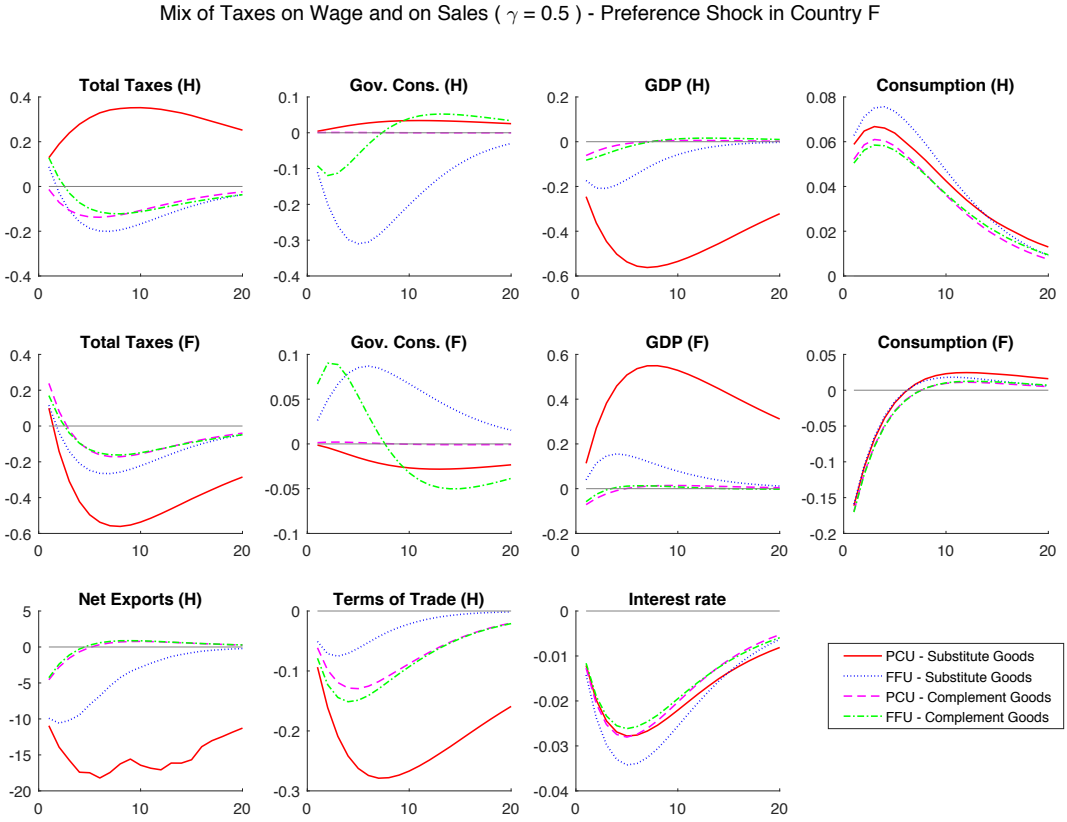
Figure 8: International Substitutes vs Complements - Technology Shock in Country H



GDP when international goods are complements and the greater difference between the two scenarios when international goods are substitutes, hold if we consider the effects of a negative preference shock in country F. After the shock, when international goods are substitutes, prices in the the monetary union decrease more in the Full Fiscal Union scenario, because of the coordination between taxes, which also makes consumption in country H increase more. Although prices in the monetary union move more, GDP is more stabilized because the fluctuations in the terms of trade and net exports are smaller. This result, however, does not apply to the case of international goods as complements, as there is very little difference in the dynamics of taxes between the two scenarios and the terms of trade are even more amplified in the Full Fiscal Union scenario.

The domination of the income effect on the substitution effect, when international goods are complements, could also affect the welfare analysis, both in terms of Consumption Equivalent Variations and in terms of an ad hoc loss function. When the traded goods are complements, net exports are mildly driven by relative

Figure 9: International Substitutes vs Complements - Preference Shock in Country F



prices, because consumers substitute less between domestic and foreign goods, and targeting net exports implies a welfare cost.

As Table 4 reports, the highest welfare gains in terms of Consumption Equivalent Variations are attained in the Pure Currency Union scenario and, since the income effect dominates the substitution effect, the largest welfare gains from output stabilization are for country H. Even if there is a welfare cost on average from stabilizing the net exports gap with a low elasticity of substitution, country H has a welfare gain from stabilizing the net exports gap with respect to the case of exogenous government consumption. When international goods are complements, the substitution effect is almost absent and stabilizing net exports makes the terms of trade more volatile. Since country F has stickier prices and greater home bias, facing higher fluctuations in relative prices without the possibility of adjusting the consumption baskets is very costly for households in this country. Finally, Table 4 shows that the consolidation of budget constraints leads to welfare losses, also when the elasticity of substitution is low. This is mainly given by the fact that the

dynamics of taxes in country F are not affected by the consolidation of budget constraints, while smoothing international spillovers amplifies the dynamics of terms of trade, generating a welfare cost for country F and a small welfare gain for country H.

Table 4: Optimal Fiscal Policy Parameters and Welfare Costs based on CEV - Complements

Policy Scenarios	Optimal Parameters*		Conditional Welfare Costs		
	ψ	ψ^*	Country H	Country F	Average
PCU (exogenous)	0	0	0%	0%	0%
PCU	0.038	0.103	-2.051%	-0.245%	-0.968%
CCU	0.083	0.084	-0.039%	0.279%	0.15%
FFU	0.083	0.084	-0.019%	0.304%	0.175%
FFU (exogenous)	0	0	-0.019%	0.304%	0.175%

*The optimal parameters have been selected by maximizing the unconditional expectation of lifetime utility.

Since there is no trade-off between price and output stabilization if the international goods are complements, fiscal policy is able to successfully stabilize the output gap. For this reason, Table 5 reports that only in the Pure Currency Union scenario with countercyclical government consumption there is a gain in terms of an ad hoc loss function, compared to the case of exogenous government consumption. The intuition behind this finding is that targeting net exports, when the international trade elasticity is low, implies a higher volatility in government consumption, which increases the volatility in prices. Since net exports move in the opposite direction with respect to the terms of trade, government consumption has the opposite effect on the terms of trade with respect to the case of international substitutes. Thus, the fluctuations in relative prices are larger, but the overall volatility of output is barely affected. Finally, consolidating budget constraints partially offsets the amplification effect due to the stabilization of net exports, which is why the Full Fiscal Union scenario, despite being less stabilizing than the Pure Currency Union scenario, has a lower cost in terms of the ad hoc loss function than the Coordinated Currency Union scenario, as we can see in Table 5.

Table 5: Welfare Gains based on an ad hoc Loss Function - Complements

Policy Scenarios	Losses			Welfare Gains*		
	Country H	Country F	Average	Country H	Country F	Average
PCU (exogenous)	0.0727	0.0634	0.0671	0%	0%	0%
PCU	0.0727	0.0630	0.0669	0%	0.63%	0.36%
CCU	0.1249	0.2479	0.1987	-71.80%	-291.01%	-196.04%
FFU	0.0812	0.1034	0.0945	-11.69%	-63.09%	-40.82%
FFU (exogenous)	0.0858	0.0723	0.0777	-18.02%	-14.04%	-15.76%

* Welfare Gains are computed as $\frac{Loss_b - Loss_a}{Loss_b}$, with $Loss_b$ the loss in the PCU with $\psi = \psi^* = 0$.

7 Conclusions and Possible Extensions

This paper provides a characterization of the stabilization properties of different fiscal policy scenarios in a Two-Country Open-Economy New-Keynesian DSGE model of a Currency Union, calibrated on the European Economic and Monetary Union. We consider three fiscal policy scenarios. In the Pure Currency Union scenario, each fiscal authority moves government spending countercyclically in response to deviations of real output from steady state. In the other two scenarios – Coordinated Currency Union and Full Fiscal Union respectively – each government targets procyclically a common variable, the net exports gap, but additionally in the Full Fiscal Union scenario the two government budget constraints are consolidated and the tax rates in the two countries move together to finance the government expenditure. In all three scenarios, the fiscal authorities have access only to distortionary taxation and must balance the budget. The presence of distortionary taxation and nominal price rigidity implies that fiscal policy and monetary policy are interconnected and produce real effects in the economy.

The main difference between our work and the previous literature is that we consider jointly the stabilization properties of targeting rules for fiscal policy and its financing with distortionary taxation, while most literature assumes stochastic government spending and lump-sum taxes, which imply very different dynamics. Based on the strength of the targeting rule, distortionary taxation might reverse the qualitative effects of fiscal policy on the economy, bringing to very different results.

Our main result is that coordinating fiscal policy by targeting the net exports gap produces much more stabilization in the economy than targeting the output gap, which provides even less stabilization on output itself. By also consolidating budget constraints we attain the most stabilized dynamics between countries through the joint adjustment of the tax rates. Actually, even only consolidating budget constraints with stochastic government consumption produces about as much stabilization as only targeting the net exports gap without consolidating budget constraints, which implies that adding one dimension to the other barely produces more stabilization. This can be viewed as two instruments for stabilization that can be used jointly, while one can make up for the temporary lack of the other, as a sort of insurance mechanism.

Our analysis also highlights that taxes on labour income entail exponentially more distortionary effects than taxes on firm sales, which implies that the latter should be generally preferred to the former. More specifically, we find that the volatility of output and the terms of trade is increasing in the share of labour income taxes over total taxation, while volatility of inflation is reduced only when taxation is lump-sum.

We also show that if the international goods are considered as complements, instead of substitutes, most of the effects of shocks are less amplified, with reversed dynamics of net exports, consistently with the previous literature. There is also much also less difference in the dynamics among the different scenarios,

when international goods are complements. The Pure Currency Union scenario performs better than all other scenarios when international goods are complements, because the substitution effect is almost absent, so that fluctuations in net exports do not offset the stabilization properties of responding to real output. As in the case of international goods as substitutes, the Full Fiscal Union scenario improves the stability of the economy with respect to the Coordinated Currency Union one, which is actually destabilizing with international goods as complements, compared to the Pure Currency Union scenario. Consolidating budget constraints decreases international spillovers between the two countries both in the case of international goods as complements and substitutes.

The policy prescriptions of our research are that countries in the Eurozone should reduce international demand imbalances by either stabilizing trade flows across countries or by creating some form of fiscal union with a common budget and taxation strategy, or both, while avoiding the excessive use of labour taxes, in favour of sales taxes. This would dampen most of the shocks hitting Eurozone countries by reducing international spillovers. A future fiscal union would also need a Eurozone treasury for collective decision-making on fiscal policy, which would need to be accountable and legitimated democratically, but this is out of the scope of the present paper.

Our model has nonetheless some assumptions that can be modified to assess different transmission mechanisms of fiscal policy which also entail possible future avenues of research. If fiscal policy has a cross-country insurance role with *complete* international financial markets, it could have an even stronger insurance role with *incomplete* international financial markets, where private cross-country insurance is not available. We do not take into consideration explicitly the zero lower bound on the nominal interest rate, which is an important feature of the recent global liquidity trap. In this case too, if fiscal policy affects the economy when there are no constraints on monetary policy, with this constraint fiscal policy might be even more effective. Last but not least, we only consider the case of balanced budget policies for fiscal authorities, so it could be interesting to study the effects of fiscal rules which allow for government debt to vary over time.

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A Mathematical Appendix

Here we provide all the equilibrium conditions and steady state values of the model.

A.1 Equilibrium Conditions

Here we collect all the equilibrium conditions of the full model, differentiating between a pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union.

The equilibrium conditions of the model are grouped into the following blocks:

Aggregate Demand Block

The aggregate demand block is composed of aggregate demand in both countries H:

$$Y_t = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[1-\alpha+\alpha^*\frac{1-h}{h}\left(\frac{\xi_t^*}{\xi_t}\right)^{\frac{1}{\sigma}}(\mathcal{S}_t)^{\eta-\frac{1}{\sigma}}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1-\eta\sigma}{\sigma(\eta-1)}}\right] C_t + G_t \tag{A.1.1}$$

and F:

$$Y_t^* = \left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{\eta}{1-\eta}} \left[1-\alpha^*+\alpha\frac{h}{1-h}\left(\frac{\xi_t}{\xi_t^*}\right)^{\frac{1}{\sigma}}(\mathcal{S}_t)^{\frac{1}{\sigma}-\eta}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1-\eta\sigma}{\sigma(1-\eta)}}\right] C_t^* + G_t^* \tag{A.1.2}$$

while the evolution of private consumption is given by the households' Euler Equation in country F:

$$\frac{1}{1+i_t} = \beta E_t\left\{\frac{\xi_{t+1}^*}{\xi_t^*}\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\frac{1}{\Pi_{t+1}^*}\right\} \tag{A.1.3}$$

and by the international risk-sharing condition in country H:

$$C_t = \frac{h}{1-h}\left[\frac{\xi_t}{\xi_t^*}\mathcal{S}_t\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{1}{1-\eta}}\right]^{\frac{1}{\sigma}} C_t^* \tag{A.1.4}$$

while the relationship between CPI inflation and PPI inflation is given by:

$$\Pi_t = \Pi_{H,t}\left[\frac{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}{1-\alpha+\alpha(\mathcal{S}_{t-1})^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{A.1.5}$$

in country H and:

$$\Pi_t^* = \Pi_{H,t}^*\left[\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha^*+\alpha^*(\mathcal{S}_{t-1})^{\eta-1}}\right]^{\frac{1}{1-\eta}} \tag{A.1.6}$$

in country F, and the evolution of the terms of trade is given by:

$$\mathcal{S}_t = \frac{\Pi^*_{H,t}}{\Pi_{H,t}} \mathcal{S}_{t-1} \tag{A.1.7}$$

while the exogenous demand shocks evolve according to:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \tag{A.1.8}$$

$$\xi^*_t = (\xi^*_{t-1})^{\rho^*_\xi} e^{\varepsilon_t} \tag{A.1.9}$$

Aggregate Supply Block

The aggregate supply block is composed of the aggregate supply equation for country H:

$$\left(\frac{1 - \theta(\Pi_{H,t})^{\varepsilon - 1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K_t}{F_t} \tag{A.1.10}$$

where:

$$K_t \equiv \xi_t (C_t)^{-\sigma} Y_t MC_t + \beta\theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon+1}}{\Pi_{t+1}} K_{t+1} \right\} \tag{A.1.11}$$

$$F_t \equiv \xi_t (C_t)^{-\sigma} Y_t (1 - \tau^s_t) + \beta\theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon}}{\Pi_{t+1}} F_{t+1} \right\} \tag{A.1.12}$$

and marginal cost in country H is given by:

$$MC_t = \frac{(Y_t)^\varphi (d_t)^\varphi (C_t)^\sigma}{(1 - \tau^w_t)(A_t)^{1+\varphi}(h)^{\varphi+\sigma}} \left[1 - \alpha + \alpha (\mathcal{S}_t)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{A.1.13}$$

and the aggregate supply equation for country F:

$$\left(\frac{1 - \theta^*(\Pi^*_{H,t})^{\varepsilon - 1}}{1 - \theta^*} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K^*_t}{F^*_t} \tag{A.1.14}$$

where:

$$K^*_t \equiv \xi^*_t (C^*_t)^{-\sigma} Y^*_t MC^*_t + \beta\theta^* E_t \left\{ \frac{(\Pi^*_{H,t+1})^{\varepsilon+1}}{\Pi^*_{t+1}} K^*_{t+1} \right\} \tag{A.1.15}$$

$$F^*_t \equiv \xi^*_t (C^*_t)^{-\sigma} Y^*_t (1 - \tau^{*s}_t) + \beta\theta^* E_t \left\{ \frac{(\Pi^*_{H,t+1})^{\varepsilon}}{\Pi^*_{t+1}} F^*_{t+1} \right\} \tag{A.1.16}$$

and marginal cost in country F is given by:

$$MC_t^* = \frac{(Y_t^*)^{\varphi}(d_t^*)^{\varphi}(C_t^*)^{\sigma}}{(1-\tau_t^{*w})(A_t^*)^{1+\varphi}(1-h)^{\varphi+\sigma}}\left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{1}{1-\eta}} \tag{A.1.17}$$

while the evolution of price dispersion is given by:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^{\varepsilon} + (1-\theta)\left[\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.1.18}$$

for country H, and:

$$d_t^* = \theta^* d_{t-1}^*(\Pi_{H,t}^*)^{\varepsilon} + (1-\theta^*)\left[\frac{1-\theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1-\theta^*}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.1.19}$$

for country F, while the levels of technology evolve exogenously according to:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \tag{A.1.20}$$

$$A_t^* = (A_{t-1}^*)^{\rho_a^*} e^{\varepsilon_t} \tag{A.1.21}$$

Net Exports, Net Foreign Assets and the Balance of Payments

Real Net Exports for country H are given by:

$$\widetilde{NX}_t \equiv Y_t - \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C_t - G_t \tag{A.1.22}$$

Real Net Foreign Assets for country H are given by:

$$\widetilde{NFA}_t \equiv \tilde{D}_t + \tilde{B}_t - \tilde{B}_t^G \tag{A.1.23}$$

The real Balance of Payments for country H is given by:

$$\widetilde{BP}_t \equiv \widetilde{NX}_t + \frac{i_{t-1}}{\Pi_{H,t}}\widetilde{NFA}_{t-1} \tag{A.1.24}$$

so that real Net Foreign Assets for country H evolve according to:

$$\widetilde{NFA}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t \tag{A.1.25}$$

Monetary Policy

Monetary policy sets the nominal interest rate following the rule:

$$\beta(1+i_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} [\beta(1+i_{t-1})]^{\rho_i} \tag{A.1.26}$$

where union-wide CPI inflation is defined by:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \tag{A.1.27}$$

Fiscal Policy in a Pure Currency Union

Fiscal policy, in a Pure Currency Union scenario, sets government consumption following the rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.28}$$

for country H, and:

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.29}$$

for country F, while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \text{and} \qquad \tilde{T}_t^* = \tilde{T}^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.30}$$

so financing fiscal policy by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ ($\gamma^* \in [0,1]$ for country F) and $1-\gamma$ ($1-\gamma^*$ for country F) through the following tax rules:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{A.1.31}$$

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1-\gamma^*)(\tau_t^{*w} - \tau^{*w}) \tag{A.1.32}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{A.1.33}$$

$$G_t^* + \tilde{T}_t^* + i_{t-1}\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.34}$$

Fiscal Policy in a Coordinated Currency Union

Fiscal policy, in a Coordinated Currency Union scenario, sets government consumption following the rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.35}$$

for country H, and:

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.36}$$

for country F, while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \text{and} \qquad \tilde{T}_t^* = \tilde{T}^* \qquad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.37}$$

so financing fiscal policy by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ ($\gamma^* \in [0,1]$ for country F) and $1-\gamma$ ($1-\gamma^*$ for country F) through the following tax rules:

$$\gamma(\tau_t^s - \tau^s) = (1-\gamma)(\tau_t^w - \tau^w) \tag{A.1.38}$$

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1-\gamma^*)(\tau_t^{*w} - \tau^{*w}) \tag{A.1.39}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{A.1.40}$$

$$G_t^* + \tilde{T}_t^* + i_{t-1}\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.41}$$

Fiscal Policy in a Full Fiscal Union

Fiscal policy, in a Full Fiscal Union scenario, sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.42}$$

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.43}$$

while keeping real transfers constant and balancing the overall budget:

$$\tilde{T}_t = \tilde{T} \qquad \tilde{T}^*_t = \tilde{T}^* \qquad \text{and} \qquad \tilde{\mathcal{B}}^G_t = \frac{\tilde{\mathcal{B}}^G_{t-1}}{\Pi_{H,t}} \implies \tilde{B}^G_t + S_t \tilde{B}^{*G}_t = \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} + S_t \frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} \tag{A.1.44}$$

so financing fiscal policy by the variation of the tax rates on labour income and firm sales from their steady state levels respectively by a share $\gamma \in [0,1]$ and $1-\gamma$, while distributing equally among the two countries the cost of fiscal policy, through the following tax rules:

$$\gamma(\tau^s_t - \tau^s) = (1-\gamma)(\tau^w_t - \tau^w) \tag{A.1.45}$$

$$(\tau^s_t - \tau^s) = (\tau^{*s}_t - \tau^{*s}) \tag{A.1.46}$$

$$(\tau^w_t - \tau^w) = (\tau^{*w}_t - \tau^{*w}) \tag{A.1.47}$$

with the following consolidated budget constraint:

$$G_t + \tilde{T}_t + S_t(G^*_t + \tilde{T}^*_t) + i_{t-1}\frac{\tilde{\mathcal{B}}^G_{t-1}}{\Pi_{H,t}} = (\tau^s_t + \tau^w_t MC_t d_t)Y_t + (\tau^{*s}_t + \tau^{*w}_t MC^*_t d^*_t)S_t Y^*_t + \tilde{\mathcal{B}}^G_t - \frac{\tilde{\mathcal{B}}^G_{t-1}}{\Pi_{H,t}} \tag{A.1.48}$$

We can now define an equilibrium for the Currency Union.

Definition 1 (Equilibrium)**.** *An Imperfectly competitive equilibrium is a sequence of stochastic processes* $\mathcal{X}_t \equiv \{Y_t, Y^*_t, C_t, C^*_t, \Pi_{H,t}, \Pi^*_{H,t}, \Pi_t, \Pi^*_t, \Pi^U_t, S_t, K_t, K^*_t, F_t, F^*_t, MC_t, MC^*_t, d_t, d^*_t, \widetilde{NX}_t, \widetilde{NFA}_t, \widetilde{CA}_t\}$ *and exogenous disturbances* $\mathcal{Z}_t \equiv \{\xi_t, \xi^*_t, A_t, A^*_t\}$ *satisfying equations A.1.1 through A.1.25 and the definition of union-wide inflation A.1.27, given initial conditions* $\mathcal{I}_{-1} \equiv \{C_{-1}, C^*_{-1}, \Pi_{H,-1}, \Pi^*_{H,-1}, S_{-1}, d_{-1}, d^*_{-1}, \widetilde{NFA}_{-1}\}$*, plus monetary and fiscal policies* $\mathcal{P}_t \equiv \{i_t, G_t, G^*_t, \tilde{T}_t, \tilde{T}^*_t, \tau^s_t, \tau^{*s}_t, \tau^w_t, \tau^{*w}_t, \tilde{B}^G_t, \tilde{B}^{*G}_t\}$ *specified in equation A.1.26 for monetary policy and in equations A.1.28 through A.1.48 for the various specifications of fiscal policy, for* $t \geq 0$*.*

A.2 The Steady State

We describe the symmetric (in terms of per capita consumption and prices) non-stochastic steady state with constant government debt and zero inflation, which will be the starting point of our simulations. We focus on the perfect foresight steady state and equilibrium deviations from it, given by different shocks. *Perfect Foresight* is a viable assumption because, despite the uncertainty to which price-setters are subject, it disappears in the aggregate due to the further assumption that there is a large number (more accurately,

a continuum) of firms, as explained in Calvo (1983).

The symmetric non-stochastic steady state with constant government debt and zero inflation, which will be the starting point of our simulations, is defined by the following assumptions and equations.

All shocks are constant at their long-run levels of 1:

$$\xi = \xi^* = A = A^* = 1 \tag{A.2.1}$$

There is no inflation and no price dispersion:

$$\Pi_H = \Pi_H^* = \Pi = \Pi^* = \Pi^U = 1 \implies d = d^* = 1 \tag{A.2.2}$$

The terms of trade and the real exchange rate are equal to 1:

$$S = 1 \implies Q = 1 \tag{A.2.3}$$

Per-capita consumption is equal across countries:

$$\frac{C}{h} = \frac{C^*}{1-h} \tag{A.2.4}$$

Aggregate demand in each country is given by:

$$Y = \left(1 - \alpha + \alpha^* \frac{1-h}{h}\right) C + G \tag{A.2.5}$$

$$Y^* = \left(1 - \alpha^* + \alpha \frac{h}{1-h}\right) C^* + G^* \tag{A.2.6}$$

Combining the previous equations we can derive per-capita consumption in each country as a function of output and government spending and equate the two to derive an equation linking output and government spending in the two countries:

$$Y = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} [Y^* - G^*] + G \tag{A.2.7}$$

From the Euler Equations:

$$\frac{1}{1+i} = \beta \implies i = \frac{1}{\beta} - 1 \tag{A.2.8}$$

Recalling marginal costs in steady state from price-setting:

$$MC = \frac{\varepsilon - 1}{\varepsilon}(1 - \tau^s) \tag{A.2.9}$$

$$MC^* = \frac{\varepsilon - 1}{\varepsilon}(1 - \tau^{*s}) \tag{A.2.10}$$

Marginal costs are also given by labour market equilibrium:

$$MC = \frac{(Y)^{\varphi}(C)^{\sigma}}{(1 - \tau^w)(h)^{\varphi+\sigma}} \tag{A.2.11}$$

$$MC^* = \frac{(Y^*)^{\varphi}(C^*)^{\sigma}}{(1 - \tau^{*w})(1 - h)^{\varphi+\sigma}} \tag{A.2.12}$$

Equating the two marginal cost expressions for each country yields consumption in terms of output:

$$C = \left[\frac{\varepsilon - 1}{\varepsilon}\frac{(1 - \tau^s)(1 - \tau^w)(h)^{\varphi+\sigma}}{(Y)^{\varphi}}\right]^{\frac{1}{\sigma}} \tag{A.2.13}$$

$$C^* = \left[\frac{\varepsilon - 1}{\varepsilon}\frac{(1 - \tau^{*s})(1 - \tau^{*w})(1 - h)^{\varphi+\sigma}}{(Y^*)^{\varphi}}\right]^{\frac{1}{\sigma}} \tag{A.2.14}$$

Deriving per-capita consumption in the two countries and equating the two yields an equation linking output in the two countries:

$$Y^* = \frac{1 - h}{h}\left[\frac{(1 - \tau^{*s})(1 - \tau^{*w})}{(1 - \tau^s)(1 - \tau^w)}\right]^{\frac{1}{\varphi}} Y \tag{A.2.15}$$

In steady state real net exports are given by:

$$\widetilde{NX} = Y - C - G \tag{A.2.16}$$

while real net foreign assets are:

$$\widetilde{NFA} = \tilde{D} + \tilde{B} - \tilde{B}^G \tag{A.2.17}$$

The real balance of payments is given by:

$$\widetilde{BP} = \widetilde{NX} + \left(\frac{1}{\beta} - 1\right)\widetilde{NFA} \tag{A.2.18}$$

while from the budget constraints of households and governments, or equivalently from the evolution of net foreign assets:

$$\widetilde{NFA} = \widetilde{NFA} + \widetilde{BP} \tag{A.2.19}$$

which implies that in steady state the balance of payments must be zero and so net exports pin down net foreign assets:

$$\widetilde{BP} = 0 \implies \widetilde{NX} = -\left(\frac{1}{\beta} - 1\right)\widetilde{NFA} \tag{A.2.20}$$

These relations yield an equation linking output, government consumption and household consumption in the two countries in steady state, which mainly comes from the fact that net exports are in zero international net supply in steady state:

$$Y + Y^* = C + C^* + G + G^* \tag{A.2.21}$$

The household budget constraints in steady state for countries H and F are given by:

$$C = \left(\frac{1}{\beta} - 1\right)(\tilde{D} + \tilde{B}) + \tilde{T} + Y(1 - \tau^s - \tau^w MC\,) \tag{A.2.22}$$

$$C^* = \left(\frac{1}{\beta} - 1\right)(\tilde{D}^* + \tilde{B}^*) + \tilde{T}^* + Y^*(1 - \tau^{*s} - \tau^{*w} MC^*\,) \tag{A.2.23}$$

Instead the government budget constraints of the two countries in steady state read:

$$G + \tilde{T} + \left(\frac{1}{\beta} - 1\right)\tilde{B}^G = Y(\tau^s + \tau^w MC) \tag{A.2.24}$$

$$G^* + \tilde{T}^* + \left(\frac{1}{\beta} - 1\right)\tilde{B}^{*G} = Y^*(\tau^{*s} + \tau^{*w} MC^*) \tag{A.2.25}$$