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Government Debt Deleveraging in the EMU

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We evaluate the stabilization properties and welfare implications of different government debt deleveraging schemes and instruments for deleveraging in a currency union. In a two-country new-Keynesian DSGE model, with a debt-elastic government bond spread and incomplete international financial markets, we study the effects of government debt deleveraging, under a range of alternative shocks and under alternative scenarios for fiscal policy coordination. We find that greater stabilization is achieved by backloading the deleveraging process and when the two countries reduce the net exports gap. Moreover, taxes are a better instrument for deleveraging compared to government consumption or transfers. Our policy prescriptions for the Eurozone are to reduce government debt more gradually over time and less during recessions and to do so using distortionary taxes, while concentrating on reducing international demand imbalances.

JEL classification: H63, E63, F42, F45, E12

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Abstract

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Contents

1 Introduction

What are the effects of government debt deleveraging in a currency union? What are the different effects of deleveraging in the face of alternative shocks and using alternative instruments? Which is the best timing and fiscal policy coordination strategy for deleveraging?

Given a situation of high government debt in most EMU countries and a request by the European Commission to reduce government debt positions to 60% of GDP, finding the best way and timing for deleveraging is an important issue. As Ferrero (2009) shows, there are important welfare gains in adopting a flexible fiscal policy, because by constraining fiscal policy, deleveraging creates a trade-off between discipline and stabilization. This paper studies government debt deleveraging in a two-country currency union where financial markets are incomplete and different government debt levels determine a government bond spread[1]. Governments choose the level of government consumption and transfers, which are financed only by distortionary taxes on labour income and firm sales and by short-term government bonds. We assume that one country (country F, the Periphery) needs to reduce its public debt, using alternatively distortionary taxation, government consumption or transfers. Being part of a currency union, government debt deleveraging creates spillover effects on the other country (country H, Germany) through international financial and goods markets, mainly by moving relative prices. We analyze the stabilization properties of three fiscal policy scenarios for deleveraging, also in the presence of supply and demand shocks and in the presence of the Zero Lower Bound: in the Pure Currency Union (PCU) scenario each country stabilizes the output gap, while in the Coordinated Currency Union (CCU) scenario the two countries stabilize the net exports gap independently, and in the Full Fiscal Union (FFU) scenario, in addition to stabilizing the net exports gap, the two countries consolidate their budget constraints. Therefore, only in the FFU scenario the two countries coordinate in terms of both cyclical and deleveraging policies.

Our analysis shows that deleveraging amplifies the overall volatility of the economy. To that extent, the role of deflationary pressures induced by the deleveraging shock is crucial in its international transmission, with and without other shocks. After a deleveraging shock to country F, it is country H which falls into a recession, because foreign goods are now more convenient, lowering substantially net exports in country H due to its greater openness to trade. At the same time, GDP in country F increases, as also found in Cogan et al. (2013), mainly induced by the deflationary effect on net exports through the terms of trade. This mechanism is also underlined by Coenen, Mohr and Straub (2008) and Forni, Gerali and Pisani (2010). Thus, reducing the volatility of net exports is key for the purpose of stabilization and can be done in two different ways: when transfers are used to deleverage the FFU scenario stabilizes more, while when taxes are used to deleverage the CCU scenario creates more stabilization. In particular, the expansion in GDP in country F increases the tax base, pushing the fiscal authority to reduce the tax rates to balance the

[1] This assumption implies that the international trade of foreign bonds is subject to intermediation costs, following mainly De Paoli (2009), which in turn follows Turnovsky (1985). As explained in Ghironi (2006) and Schmitt-Grohé and Uribe (2003) this assumption ensures stationarity of net foreign assets and is an assumption also used in Eggertsson, Ferrero and Raffo (2014).

government budget. As argued by Ferrara and Tirelli (2017), if fiscal policy is able to anticipate the reduction in taxes while simultaneously cutting government spending, this highly reduces the cost of deleveraging. Because of their ability to reduce the deflation induced by the debt reduction in country F and, in turn, its effects on country H, distortionary taxes are always a better instrument for deleveraging because they stabilize the economy more than other instruments. In terms of speed, backloading the deleveraging process stabilizes the economy more by reducing deflationary pressures faster. This result is in line with Romei (2015), according to which, if the instrument used for deleveraging is taxation, public debt should be reduced slowly.

Our research is related to two different strands of literature: one that has focused on New Keynesian DSGE models and another one on government debt deleveraging. We follow the open economy approach of Galí (2009), but in a two-country setting[2] with only distortionary taxes as sources of government revenue like in Ferrero (2009). To this framework we added, in our previous work, Cole, Guerello and Traficante (2016), home bias in consumption, to allow for deviations from Purchasing Power Parity, and targeting rules for fiscal policy, to allow governments to coordinate. We now extend our previous model with a debt-elastic government bond spread and incomplete international financial markets, to account for international financial frictions following Hjortsø (2016).

This paper is also related to the literature on government debt deleveraging, which has been investigated thoroughly after the Great Recession and the sovereign debt crisis in the Eurozone. Some recent papers, such as Ferrara and Tirelli (2017), analyze the fiscal policy mix for public debt deleveraging and its consequences in a closed economy, while Almeida et al. (2013) considers a small open economy setup. However, the slump which follows a fiscal consolidation, as observed in these papers, is only partially confirmed by the literature on deleveraging in large open economies, such as Coenen, Mohr and Straub (2008), Forni, Gerali and Pisani (2010) and Cogan et al. (2013), which underline the relevance of fluctuations in net exports.

Like Coenen, Mohr and Straub (2008), our paper deals with government debt deleveraging in a currency union, and the debt reduction we treat reproduces what member countries in the Eurozone are requested to do in order to reach the target of 60% of GDP, as stated in the Maastricht Treaty. We analyze the welfare implications of different paths for dynamic government debt deleveraging and study the best timing for public debt deleveraging and the optimal taxation and spending combinations. To our knowledge, Romei (2015) is the only paper that focuses on the timing of public debt deleveraging. We contribute to this literature by taking into account the possibility of a Redemption Fund, as outlined in Van Rompuy et al. (2012). The possibility for countries in dire circumstances, as during tight fiscal consolidations, to obtain a partial debt relief through transfers

[2]The structure of Galí and Monacelli (2008) and Farhi et al. (2013) with a continuum of countries means that more variables will be exogenous, compared to a two-country model, and that a single country, being one of an infinite continuum, as specified in Galí (2009), does not influence any world variable. This means that all world variables must be exogenous and that it is harder to see the interaction among countries, so that international trade plays no role because any expenditure on goods from any one country has a value of zero, being one of infinitely many composing the integral, as written in Galí (2009). This poses questions on the validity of such a model and pushes us to prefer a two-country model instead.

by a central EMU government may be welfare improving for the union as a whole, especially in the face of asymmetric shocks.

Our model is structured to allow for spillovers from monetary to fiscal policy and viceversa, and from one country to another through country-specific fiscal policies. Nominal rigidities, in the form of staggered price adjustments, generate real effects of monetary policy, while distortionary taxation generates non-Ricardian effects of fiscal policy. This framework allows to study the interaction between country-specific fiscal policies, where in the absence of the nominal exchange rate as an automatic stabilizer, fiscal policies influence each other through their effects on output and the terms of trade. Incomplete international financial markets with a government bond spread and government debt deleveraging dynamics provide additional frictions in the economy which can be addressed by monetary or fiscal authorities, or by an additional macro-prudential authority.

The remainder of the paper is structured as follows. Section 2 describes the general model and fiscal policy scenarios (Pure Currency Union, Coordinated Currency Union and Full Fiscal Union) with different deleveraging instruments (Government Consumption, Transfers and Taxes). Section 3 shows the calibration of the parameters and steady state stances of the model to two groups of countries in the European Monetary Union: Germany and the Periphery. Section 4 provides numerical simulations under different deleveraging scenarios, comparing Pure Currency Union, Coordinated Currency Union and Full Fiscal Union outcomes, other than alternative fiscal policy instruments for deleveraging and alternative deleveraging schemes, also at the Zero Lower Bound. Section 5 describes a welfare measure based on an ad hoc loss function and provides welfare evaluations of the different deleveraging scenarios and instruments. Section 6 collects the main conclusions and provides possible extensions. Appendix A.1 collects all equilibrium conditions of the model used for the simulations, while Appendix A.2 describes the steady state on which the model is calibrated. Appendix B describes a sensitivity analysis to changes in some key parameters of the model.

2 A Two-Country Currency Union Model

The EMU is represented by two countries (or groups of countries) of different size forming a Currency Union. Both economies are assumed to share identical preferences, technology and market structure, but may be subject to different shocks, price rigidities, initial conditions and fiscal stances. The two countries are indexed by H and F for Home and Foreign. We can think of country H as Germany and country F as the rest of the Eurozone, or the Periphery. The EMU is inhabited by a continuum of infinitely-lived households of measure one, indexed by $i \in [0, 1]$. Each household owns a monopolistically competitive firm producing a differentiated good, indexed by $j \in [0, 1]$. The population on the segment $[0, h)$ belongs to country H while the population on the segment $[h, 1]$ belongs to country F, so that the relative size of country H is $h \in [0, 1]$, while the relative size of country F is $1 - h$. This is true for both households and firms. Each country has an independent Fiscal Authority, while the Currency Union shares a common Monetary Authority. Monetary policy sets the interest rate for the whole currency union following an inflation targeting regime. Fiscal policy is designed following the Fiscal Compact Rules, by imposing that the target Government Debt-to-GDP ratio is about 60% and that countries must either adopt a balanced budget law in their national legislation or adopt the Debt Brake Rule, which implies that if government debt-to-GDP is more than 60% it should decrease by 5% of the excess every year. In our model there is a stabilization rule for fiscal policy, in a setting where one country has to deleverage, while the other country balances the budget.

In describing the model and monetary and fiscal policies in detail in what follows, we denote variables referred to the *Foreign* country with a star (*) and, given symmetry between the two countries, we show the main equations only for country H, while we also show the equations for country F only when they differ.

2.1 Households

In each country there is a continuum of households, which gain utility from private consumption and disutility from labour, consume goods produced in both countries with home bias, supply labour to domestic firms, and collect profits from those firms. Households can trade a complete set of one-period state-contingent claims only within their own country. Households in country H can purchase one-period bonds issued by both countries' governments, while households in country F can only purchase one-period bonds issued by their own country's government, subject to their budget constraints. This reflects the fact that Germany (Country H) is a net creditor on international financial markets, while the rest of the Eurozone (the Periphery, country F) is a net debtor on international financial markets.

Each household in country H, indexed by $i \in [0, h)$ seeks to maximize the present-value utility[3]:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \frac{(C_t^i)^{1-\sigma} - 1}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \tag{2.1.1}$$

where $\beta \in [0, 1]$ is the common discount factor, which households use to discount future utility, σ is the inverse of the elasticity of intertemporal substitution (it is also the Coefficient of Relative Risk Aversion (CRRA)), φ is the inverse of the Frisch elasticity of labour supply and ξ_t is a preference shock to *Home* households. This preference shock is assumed to follow the AR(1) process in logs:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \tag{2.1.2}$$

where $\rho_\xi \in [0, 1]$ is a measure of persistence of the shock and ε_t is a zero mean white noise process. N_t^i denotes hours of labour supplied by households in country H. C_t^i is a composite index for private

[3] We choose to specify additively separable period utility of the type with Constant Relative Risk Aversion (CRRA), so with constant elasticity of intertemporal substitution, and with constant elasticity of labour supply.

consumption defined by:

$$C_t^i \equiv \left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t}^i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t}^i)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.1.3}$$

for households in country H, while the analogous index for households in country F, C_t^{*i}, is defined by:

$$C_t^{*i} \equiv \left[(1-\alpha^*)^{\frac{1}{\eta}}(C_{H,t}^{*i})^{\frac{\eta-1}{\eta}} + \alpha^{*\frac{1}{\eta}}(C_{F,t}^{*i})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2.1.4}$$

where the parameter $\eta > 0$ measures the substitutability between domestic and foreign goods (international trade elasticity), and $\alpha \in [0,1]$ is a measure of openness of the *Home* economy to international trade. Equivalently $(1-\alpha)$ is a measure of the degree of home bias in consumption in country H. When α tends to zero the share of foreign goods in domestic consumption vanishes and the country ends up in autarky, consuming only domestic goods. If $1-\alpha > h$ there is home bias in consumption in country H, because the share of consumption of domestic goods is greater than the share of production of domestic goods. The same applies to the *Foreign* parameter of openness to international trade $\alpha^* \in [0,1]$ for country F, except for the fact that if $1-\alpha^* > 1-h$ there is home bias in consumption in country F. $C_{H,t}^i$ is an index of consumption of domestic goods for households in country H, given by the constant elasticity of substitution (CES) function (also known as Dixit and Stiglitz (1977) aggregator function):

$$C_{H,t}^i \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.1.5}$$

whereas, for households in country F the same index of consumption of domestic goods, $C_{H,t}^{*i}$, is given by:

$$C_{H,t}^{*i} \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{H,t}^{*i}(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.1.6}$$

where $j \in [0,1]$ denotes a single good variety of the continuum of differentiated goods produced in the world economy and the parameter $\varepsilon > 1$ measures the elasticity of substitution between varieties produced within a given country. $C_{F,t}^i$ is an index of imported goods for households in country H, given by the analogous CES function:

$$C_{F,t}^i \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{F,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.1.7}$$

while the same index for imported goods for households in country F, $C_{F,t}^{*i}$, is given by:

$$C_{F,t}^{*i} \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{F,t}^{*i}(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.1.8}$$

Households in country H maximize their present-value utility, equation 2.1.1, subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}(j)C^i_{H,t}(j)\,dj + \int_h^1 P_{F,t}(j)C^i_{F,t}(j)\,dj + D^i_t + B^i_{H,t} + B^i_{F,t}$$
$$\leq \frac{D^i_{t-1}}{\mathcal{Q}_{t-1,t}} + B^i_{H,t-1}(1+i_{t-1}) + B^i_{F,t-1}(1+i^*_{t-1})(1-\delta_{t-1}) + (1-\tau^w_t)W_tN^i_t + T^i_t + \Gamma^i_t + \mathcal{I}^{*i}_t \tag{2.1.9}$$

for $t = 0, 1, 2, \ldots$, where $P_{H,t}(j)$ is the price of domestic variety j, $P_{F,t}(j)$ is the price of variety j imported from country F, D^i_{t-1} is the portfolio of state-contingent claims purchased by the household in period $t-1$, $\mathcal{Q}_{t-1,t}$ is the stochastic discount factor for households in country H, which is different for households in the two countries and represents the price of state-contingent claims or equivalently the inverse of the gross return on state-contingent claims, $B^i_{H,t}$ are government bonds issued by country H and purchased by the household in period t, i_{t-1} is the nominal interest rate set by the central bank in period $t-1$, which is also the net return on government bonds issued by country H, W_t is the nominal wage for households in country H, T^i_t denotes lump-sum transfers from the government to households, Γ^i_t denotes the share of profits net of taxes to households from ownership of firms, $\mathcal{I}^i_t$ denotes the share of profits to households in country H from ownership of the financial intermediaries and $\tau^w_t \in [0,1]$ is a marginal tax rate on labour income paid by households to the government.

The only financial assets traded internationally are given by $B^i_{F,t}$, which are government bonds issued by country F and purchased by households in country H in period t, i^*_{t-1} is the nominal interest rate for country F in period $t-1$, which is also the net return on government bonds issued by country F, while $\delta_t \in [0,1]$ is a transaction cost for households in country H on purchases of government bonds issued by country F, given by the AR(1) process:

$$\delta_t \equiv (1-\rho_\delta)\delta^B\left(\frac{B^{*G}_{t-1}}{P^*_{H,t-1}Y^*_{t-1}} - \frac{B^{*G}}{P^*_HY^*}\right) + \rho_\delta\delta_{t-1} \tag{2.1.10}$$

where $\frac{B^{*G}_{t-1}}{P^*_{H,t-1}Y^*_{t-1}}$ is the overall real government debt-to-GDP for country F in period $t-1$, variables without subscript t are their respective steady state values, and $\rho_\delta \in [0,1]$ is a measure of persistence of the government bond spread shock. The previous equation shows how the transaction cost for households in country H on purchases of government bonds issued by country F, or the government bond spread given by $(1+i^*_t)\delta_t$, is increasing in the deviation of government debt-to-GDP from steady state by a factor given by δ^B, which represents the sensitivity of the government bond spread or of the transaction cost to deviations of government debt-to-GDP from steady state. This reflects a positive interest rate spread as a risk premium paid on bonds issued by a country which has a government debt/GDP ratio higher than the 60% target.

The financial intermediaries, owned by the households in country H, earn profits on all the internationally traded bonds $B^i_{F,t-1}$ by collecting savings from households in country H at the interest rate set by the central bank i_{t-1} and lending to the government in country F at the interest rate paid on its government bonds i^*_{t-1}. The aggregate profits of these financial intermediaries are then given by:

$$\mathcal{I}_t \equiv \int_0^h \mathcal{I}^i_t \, di \equiv h\mathcal{I}^i_t \equiv B_{F,t-1}\left[(1+i^*_{t-1}) - (1+i^*_{t-1})(1-\delta_{t-1})\right] = B_{F,t-1}(1+i^*_{t-1})\delta_{t-1} \quad (2.1.11)$$

where $B_{F,t-1} \equiv \int_0^h B^i_{F,t-1}\, di \equiv hB^i_{F,t-1}$ are aggregate bonds issued by the government in country F and purchased by households in country H and where the government bond spread for country F, on which financial intermediaries make profits, is given by $(1+i^*_{t-1})\delta_{t-1}$.

Households in country F analogously maximize their present-value utility subject to the following sequence of budget constraints:

$$\begin{aligned}\int_0^h P^*_{H,t}(j)C^{*i}_{H,t}(j)\, dj + \int_h^1 P^*_{F,t}(j)C^{*i}_{F,t}(j)\, dj + D^{*i}_t + B^{*i}_{F,t}& \\ \leq \frac{D^{*i}_{t-1}}{\mathcal{Q}^*_{t-1,t}} + B^{*i}_{F,t-1}(1+i^*_{t-1}) + (1-\tau^{*w}_t)W^*_t N^{*i}_t + T^{*i}_t + \Gamma^{*i}_t& \quad (2.1.12)\end{aligned}$$

for $t = 0, 1, 2, \ldots,$ where all the starred (*) variables are the *Foreign* equivalent of the unstarred ones explained above, so that $\mathcal{Q}^*_{t-1,t}$ is the stochastic discount factor for households in country F and $B^{*i}_{F,t}$ are government bonds issued by country F and purchased by the household in country F in period t. As shown by the budget constraint, households in country F can only purchase government bonds issued by their own country and, differently from households in country H, they do not pay any transaction costs on the purchase of these government bonds.

All variables are expressed in units of the union's currency. Last but not least, households in country H are subject to the following solvency constraint, for all t, that prevents them from engaging in Ponzi-schemes:

$$\lim_{T\to\infty} E_t\left\{\mathcal{Q}_{t,T}D^i_T\right\} \geq 0 \quad (2.1.13)$$

Aggregating the intratemporal optimality condition yields the aggregate labour supply equation for households in country H:

$$N_t = (h)^{1+\frac{\sigma}{\varphi}}(C_t)^{-\frac{\sigma}{\varphi}}\left[(1-\tau^w_t)\frac{W_t}{P_t}\right]^{\frac{1}{\varphi}} \quad (2.1.14)$$

where N_t is aggregate labour supply and C_t is aggregate consumption for households in country H, defined by:

$$N_t \equiv \int_0^h N^i_t\, di = hN^i_t \qquad C_t \equiv \int_0^h C^i_t\, di = hC^i_t \quad (2.1.15)$$

while aggregating the intertemporal optimality condition for households in country H, taking conditional expectations on both sides of the equation and using the no-arbitrage condition between government bonds and state-contingent claims yields:

$$\frac{1}{(1+i_t^*)(1-\delta_t)} = \frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} = \beta E_t \left\{ \frac{\xi_{t+1}}{\xi_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} \tag{2.1.16}$$

where $\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\}$ is the price of a one-period government bond issued by country H paying off one unit of the union's currency in $t+1$ and $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is gross CPI inflation in country H.

For households in country F, taking conditional expectations on both sides of the aggregate intertemporal optimality condition and using the no-arbitrage condition between government bonds and state-contingent claims yields:

$$\frac{1}{1+i_t^*} = E_t\{\mathcal{Q}^*_{t,t+1}\} = \beta E_t \left\{ \frac{\xi^*_{t+1}}{\xi^*_t} \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\sigma} \frac{1}{\Pi^*_{t+1}} \right\} \tag{2.1.17}$$

where $\frac{1}{1+i_t^*} = E_t\{\mathcal{Q}^*_{t,t+1}\}$ is the price of a one-period government bond issued by country F paying off one unit of the union's currency in $t+1$ and $\Pi^*_{t+1} \equiv \frac{P^*_{t+1}}{P^*_t}$ is gross CPI inflation in country F.

Aggregating the budget constraints of households in countries H and F respectively and considering that in optimality they hold with equality yields:

$$P_tC_t + D_t + B_{H,t} + B_{F,t} = (1+i_{t-1})(D_{t-1} + B_{H,t-1} + B_{F,t-1}) + (1-\tau_t^w)W_tN_t + T_t + \Gamma_t + \mathcal{I}_t \tag{2.1.18}$$

$$P_t^*C_t^* + D_t^* + B^*_{F,t} = \frac{1+i_{t-1}}{1-\delta_{t-1}}(D^*_{t-1} + B^*_{F,t-1}) + (1-\tau_t^{*w})W_t^*N_t^* + T_t^* + \Gamma_t^* \tag{2.1.19}$$

where aggregate contingent claims, aggregate bonds, aggregate transfers and aggregate profits are defined analogously to aggregate consumption and labour.

Since one-period state-contingent claims can be traded freely between households only within each country, they are in zero national net supply, so that the market clearing conditions for these assets in every period t are consequently given by:

$$\int_0^h D_t^i \, di = hD_t^i = D_t = 0 \qquad \int_h^1 D_t^{*i} \, di = (1-h)D_t^{*i} = D_t^* = 0 \tag{2.1.20}$$

2.2 Prices and International Assumptions

Several international identities and assumptions need to be spelled out in order to link the *Home* economy to the *Foreign* one and to be able to close the model. The Consumer Price Index (CPI) for country H is given by:

$$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{2.2.1}$$

while the Consumer Price Index (CPI) for country F is given by:

$$P_t^* \equiv \left[(1-\alpha^*)(P^*_{H,t})^{1-\eta} + \alpha^*(P^*_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{2.2.2}$$

where $P_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country H and $P_{F,t}$ is a price index for goods imported from country F, respectively defined by:

$$P_{H,t} \equiv \left(\frac{1}{h} \int_0^h P_{H,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{2.2.3}$$

$$P_{F,t} \equiv \left(\frac{1}{1-h} \int_h^1 P_{F,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{2.2.4}$$

while $P^*_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country F and $P^*_{F,t}$ is a price index for goods imported from country H, respectively defined by:

$$P^*_{H,t} \equiv \left(\frac{1}{1-h} \int_h^1 P^*_{H,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{2.2.5}$$

$$P^*_{F,t} \equiv \left(\frac{1}{h} \int_0^h P^*_{F,t}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}} \tag{2.2.6}$$

Although deviations from *Purchasing Power Parity* (PPP) may arise because of home bias in consumption, we assume that the *Law of One Price* (LOP) holds for every single good j, which implies:

$$P_{H,t}(j) = P^*_{F,t}(j) \qquad \text{and} \qquad P_{F,t}(j) = P^*_{H,t}(j) \tag{2.2.7}$$

for all $j \in [0,1]$, where $P_{H,t}(j)$ (or $P_{F,t}(j)$ for goods imported from country F) is the price of good j in country H and $P^*_{F,t}(j)$ (or $P^*_{H,t}(j)$ for goods produced in country F) is the price of good j in country F in terms of the union's currency. Plugging the previous expressions into the definitions of $P_{H,t}$ and $P_{F,t}$, equations 2.2.3 and 2.2.4, respectively yields:

$$P_{H,t} = P^*_{F,t} \qquad \text{and} \qquad P_{F,t} = P^*_{H,t} \tag{2.2.8}$$

The *terms of trade* are defined as the price of foreign goods in terms of home goods, for households in country H and in country F, and are given respectively by:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} \qquad \text{and} \qquad \mathcal{S}^*_t \equiv \frac{P^*_{F,t}}{P^*_{H,t}} \tag{2.2.9}$$

Combining the previous result with the definition of the terms of trade for countries H and F yields:

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}} = \frac{P^*_{H,t}}{P^*_{F,t}} = \frac{1}{\mathcal{S}^*_t} \tag{2.2.10}$$

The relationship between PPI inflation and CPI inflation in country H is given by:

$$\Pi_t = \Pi_{H,t} \left[\frac{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}{1-\alpha+\alpha(\mathcal{S}_{t-1})^{1-\eta}} \right]^{\frac{1}{1-\eta}} \tag{2.2.11}$$

while dividing the terms of trade in period t by the terms of trade in period $t-1$ yields a relationship showing the evolution of the terms of trade over time:

$$\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}} = \frac{\Pi^*_{H,t}}{\Pi_{H,t}} \implies \mathcal{S}_t = \frac{\Pi^*_{H,t}}{\Pi_{H,t}}\mathcal{S}_{t-1} \tag{2.2.12}$$

as a function of PPI inflation in both countries H and F.

The *Real Exchange Rate* between the *Home* country and country F is the ratio of the two countries' CPIs, expressed both in terms of the union's currency, and is defined by:

$$Q_t \equiv \frac{P^*_t}{P_t} = \mathcal{S}_t \left[\frac{1-\alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{2.2.13}$$

where the difference between the real exchange rate and the terms of trade is given by the degree of openness of the two countries and the international trade elasticity. If the countries both have complete home bias ($\alpha = \alpha^* = 0$), then they are in autarky and the real exchange rate is exactly equal to the terms of trade, because the CPI and PPI are the same in each country. Based on our calibration ($\alpha = 0.52 < 1 - h = 0.6$), the real exchange rate increases when the terms of trade increase, meaning they both move in the same direction, so we can just track the movements of the terms of trade, without having to keep track of the movements in the real exchange rate.

Since state-contingent claims cannot be traded internationally there is no full international risk-sharing, but the only assets traded internationally, the bonds issued by the government in country F, yield an equation linking the interest rates in the two countries, through the transaction cost or government bond spread, given by:

$$1 + i_t = (1 + i^*_t)(1 - \delta_t) \implies 1 + i^*_t = \frac{1 + i_t}{1 - \delta_t} \tag{2.2.14}$$

which implies that the interest rate paid on government bonds issued by country F is increasing in the transaction cost δ_t, or in the government bond spread $(1 + i^*_t)\delta_t$, other than increasing in the interest rate set by the central bank and paid on government bonds issued by country H, i_t.

2.3 Net Exports, Net Foreign Assets and the Balance of Payments

Net Exports are defined as domestic production minus domestic consumption, which is equal to exports minus imports, and for country H are given in real terms (divided by $P_{H,t}$), by:

$$\widetilde{NX}_t = Y_t - \frac{P_t}{P_{H,t}}C_t - G_t = Y_t - \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C_t - G_t \tag{2.3.1}$$

where net exports are shown to be a function of the country's degree of openness and the terms of trade, other than domestic production and public and private domestic consumption.

Net Foreign Assets are given by the sum of private and public assets held abroad, and for country H are given in real terms (divided by $P_{H,t}$), by:

$$\widetilde{NFA}_t \equiv \tilde{B}_{F,t} \qquad \text{and} \qquad \widetilde{NFA}_t^* \equiv \tilde{B}_{F,t}^* - \tilde{B}_t^{*G} \tag{2.3.2}$$

The *Balance of Payments* is given by net exports plus interest accrued on net foreign assets and income from abroad (from financial intermediaries), which in real terms (divided by $P_{H,t}$) can be written as:

$$\widetilde{BP}_t \equiv \widetilde{NX}_t + i_{t-1}\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \delta_t(1+i_{t-1}^*)\frac{B_{F,t-1}}{\Pi_{H,t}} = \widetilde{NX}_t + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} \tag{2.3.3}$$

Since government bonds issued by country H are not traded internationally, while government bonds issued by country F are traded internationally, the market clearing conditions for these bonds are given by:

$$B_{H,t} - B_t^G = 0 \qquad B_{F,t} + B_{F,t}^* - B_t^{*G} = 0 \tag{2.3.4}$$

From the households' budget constraint, substituting in firm profits and labour income, the expression for transfers backed out from the government budget constraint, the market clearing condition for government bonds issued by country H, the definitions of net exports, net foreign assets and the balance of payments, yields a relationship between net foreign assets, net exports and the balance of payments for country H, which in real terms (divided by $P_{H,t}$) can be rewritten as:

$$\widetilde{NFA}_t = \frac{1+i_{t-1}}{1-\delta_{t-1}}\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{NX}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t \tag{2.3.5}$$

Notice that all variables with a tilde ($\tilde{}$) are in real terms (divided by $P_{H,t}$).

2.4 Firms

In country H there is a continuum of firms indexed by $j \in [0, h)$, each producing a differentiated good with the same technology represented by the following production function:

$$Y_t(j) = A_t N_t(j) \tag{2.4.1}$$

where A_t represents the level of technology in country H, which evolves exogenously over time following the AR(1) process in logs:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \tag{2.4.2}$$

where $\rho_a \in [0, 1]$ is a measure of persistence of the shock and ε_t is a zero mean white noise process.

From the production function we can derive labour demand for individual firms in country H and the nominal and real marginal costs of production, which are equal across firms in each country and are given by:

$$N_t(j) = \frac{Y_t(j)}{A_t} \implies MC_t^n = \frac{W_t}{A_t} \implies MC_t = \frac{W_t}{A_t P_{H,t}} \tag{2.4.3}$$

Aggregating individual labour demand across firms in each country yields the aggregate labour demand for country H:

$$N_t \equiv \int_0^h N_t(j)\,dj = \int_0^h \frac{Y_t(j)}{A_t}\,dj = \frac{Y_t}{A_t}\int_0^h \frac{1}{h}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj = \frac{Y_t}{A_t} d_t \tag{2.4.4}$$

where Y_t and Y_t^* are aggregate output in countries H and F, respectively given by:

$$Y_t \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad Y_t^* \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 Y_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.4.5}$$

and where the terms:

$$d_t \equiv \int_0^h \frac{1}{h}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} dj \qquad \text{and} \qquad d_t^* \equiv \int_h^1 \frac{1}{1-h}\left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} dj \tag{2.4.6}$$

represent relative price dispersion across firms in each country. In steady state and in a flexible price equilibrium these relative price dispersions are equal to one.

Aggregating over all $j \in [0, h)$ Firm j's period t profits net of taxes in country H, substituting in labour demand, marginal costs, the demand function for output, using the definition of $P_{H,t}$, and substituting in price dispersion yields aggregate profits net of taxes in country H:

$$\Gamma_t = (1-\tau_t^s)P_{H,t}Y_t - P_{H,t}MC_tY_td_t = P_{H,t}Y_t(1-\tau_t^s - MC_td_t) \tag{2.4.7}$$

where τ_t^s is the marginal tax rate on firm sales in country H.

Following Calvo (1983), each firm in country H may reset its price with probability $1-\theta$ in any given period. Thus, each period a fraction $1-\theta$ of randomly selected firms reset their price, while a fraction θ keep their prices unchanged. As a result, the average duration of a price in country H is given by $(1-\theta)^{-1}$, and θ can be seen as a natural index of price stickiness for country H. In country F each firm may reset its price with probability $1-\theta^*$ in any given period. Thus, each period a fraction $1-\theta^*$ of randomly selected firms reset their price, while a fraction θ^* keep their prices unchanged. As a result, the average duration of a price in country F is given by $(1-\theta^*)^{-1}$, and θ^* can be seen as a natural index of price stickiness for country F. This allows for the two countries to have different degrees of price rigidity.

A firm in country H re-optimizing in period t will choose the price $\bar{P}_{H,t}$ that maximizes the current market value of the profits net of taxes generated while that price remains effective. Formally, it solves the problem:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \mathcal{Q}_{t,t+k} Y_{t+k|t}(j) \left[(1-\tau_{t+k}^s)\bar{P}_{H,t} - MC_{t+k}^n\right]\right\} \tag{2.4.8}$$

subject to the sequence of demand constraints:

$$Y_{t+k|t}(j) = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon} \frac{Y_{t+k}}{h} \tag{2.4.9}$$

for $k = 0, 1, 2, \ldots$, where $\mathcal{Q}_{t,t+k}$ is the households' stochastic discount factor in country H for discounting k-period ahead nominal payoffs from ownership of firms, defined by:

$$\mathcal{Q}_{t,t+k} = \beta^k \frac{\xi_{t+k}}{\xi_t} \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}} \tag{2.4.10}$$

for $k = 0, 1, 2, \ldots$, and where $Y_{t+k|t}(j)$ is the output in period $t + k$ for firm j which last reset its price in period t.

The optimal price chosen by firms in country H can be expressed as a function of only aggregate variables:

$$\bar{P}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} MC^n_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} (1 - \tau^s_{t+k}) \right\}} \tag{2.4.11}$$

Notice that in the zero inflation steady state and in the flexible price equilibrium the previous equation simplifies to:

$$\bar{P}_H = \frac{\varepsilon}{(\varepsilon - 1)(1 - \tau^s)} MC^n \tag{2.4.12}$$

where MC^n is the nominal marginal cost in steady state and in the flexible price equilibrium in country H, and where the optimal price is shown to be set as a markup over nominal marginal costs.

2.5 Central Bank and Monetary Policy

The only central bank in the currency union sets monetary policy by choosing the nominal interest rate to target *union-wide* CPI inflation through a Taylor rule. We assume that the ECB cares only about inflation, as price stability is its primary objective.

Monetary policy follows an Inflation Targeting regime of the kind:

$$\beta(1 + i_t) = \left(\frac{\Pi^U_t}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} [\beta(1 + i_{t-1})]^{\rho_i} \tag{2.5.1}$$

where union-wide inflation is defined as the population-weighted geometric average of the CPI inflations in the two countries:

$$\Pi^U_t \equiv (\Pi_t)^h (\Pi^*_t)^{1-h} \tag{2.5.2}$$

while variables without subscripts t denote their respective steady state levels, ϕ_π represents the responsiveness of the interest rate to inflation and ρ_i is a measure of the persistence of the interest rate over time (interest rate smoothing).

In the alternative scenario it has a Zero Lower Bound (ZLB) constraint on its policy rate, the nominal interest rate: $i_t \geq 0$, which makes the rule become:

$$i_t = \max\{\tilde{i}_t, 0\} \tag{2.5.3}$$
$$\beta(1+\tilde{i}_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} \left[\beta(1+\tilde{i}_{t-1})\right]^{\rho_i}$$

2.6 Government and Fiscal Policy in a Pure Currency Union

As mentioned above, national governments choose the level of government consumption and transfers, which are financed by distortionary taxes on labour income and firm sales and by short-term government bonds. In a Pure Currency Union (uncoordinated fiscal policy) each government chooses the amount of government consumption or transfers for domestic stabilization purposes, by setting them to target the output gap, while using the other to deleverage government debt (or keeping it at steady state) and using a mix of distortionary tax rates on labour income and firm sales to finance remaining fiscal policy or to finance also deleveraging. In this case Germany balances its budget and the Eurozone Periphery deleverages its government debt, managing fiscal policy independently without cooperating, because they only care about stabilizing their own domestic demand.

In country H the government finances a stream of public consumption G_t and transfers T_t subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}(j)G_t(j)\,dj + \int_0^h T_t^i\,di + B_{t-1}^G(1+i_{t-1}) = B_t^G + \tau_t^s P_{H,t}Y_t + \int_0^h \tau_t^w W_t N_t^i\,di \tag{2.6.1}$$

where the right hand side represents government income from taxation and newly issued government bonds, while the left hand side represents total government spending on consumption and transfers, and on government bonds due at the end of period t, including interest. B_t^G are government bonds issued by country H in period t, while all other variables are as explained above. Government consumption, G_t, is given by the following CES function, just like equation 2.4.9 for the demand function for firms, where we assume that the government purchases only goods produced domestically (complete home bias):

$$G_t \equiv \left(\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h G_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.6.2}$$

Integrating the government budget constraint and dividing by $P_{H,t}$ yields the government budget constraint in real terms:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{2.6.3}$$

where variables with a tilde ($\tilde{}$) are in real terms (divided by $P_{H,t}$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of government debt.

In the transfer scenario, fiscal policy in country H chooses government consumption to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{2.6.4}$$

while keeping real transfers constant, as the government does not need to deleverage its government debt, and maintaining a balanced budget, through the debt rule:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \tilde{T}_t = \tilde{T} \tag{2.6.5}$$

while varying equally the tax rates on labour income and firm sales to finance the government expenditure, through the tax rule:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \tag{2.6.6}$$

where $\rho_g \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and ε_t is a zero mean white noise process. Variables without subscripts t represent their respective steady state level, while $\psi_y \geq 0$ represents the responsiveness of government consumption to variations of the output gap.

In the consumption scenario, fiscal policy in country H chooses real transfers to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{2.6.7}$$

while keeping government consumption constant, as the government does not need to deleverage its government debt, and maintaining a balanced budget, through the debt rule:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad G_t = G \tag{2.6.8}$$

while varying equally the tax rates on labour income and firm sales to finance the government expenditure, through the tax rule:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \tag{2.6.9}$$

where $\rho_t \in [0,1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while $\psi_y \geq 0$ represents the responsiveness of real transfers to variations of the output gap.

In the distortionary tax scenario, fiscal policy in country H chooses government consumption to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{2.6.10}$$

while keeping real transfers constant and maintaining a balanced budget, through the debt rule:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \tilde{T}_t = \tilde{T} \tag{2.6.11}$$

while varying equally the tax rates on labour income and firm sales to finance the government expenditure, through the tax rule:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \tag{2.6.12}$$

In country F the government finances a stream of public consumption G_t^* and transfers T_t^* subject to the following sequence of budget constraints:

$$\int_h^1 P_{H,t}^*(j)G_t^*(j)\,dj + \int_h^1 T_t^{*i}\,di + B_{t-1}^{*G}(1+i_{t-1}^*) = B_t^{*G} + \tau_t^{*s}P_{H,t}^*Y_t^* + \int_h^1 \tau_t^{*w}W_t^*N_t^{*i}\,di \tag{2.6.13}$$

where the right hand side represents government income from taxation and newly issued government bonds, while the left hand side represents total government spending on consumption and transfers, and on government bonds due at the end of period t, including interest. B_t^{*G} are government bonds issued by country F in period t, while all other variables are as explained above. Government consumption, G_t^*, is given by the following CES function, where we assume that the government purchases only goods produced domestically (complete home bias):

$$G_t^* \equiv \left(\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 G_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.6.14}$$

Integrating the government budget constraint and dividing by $P_{H,t}^*$ yields the government budget constraint in real terms:

$$G_t^* + \tilde{T}_t^* + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s}Y_t^* + \tau_t^{*w}MC_t^*d_t^*Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{2.6.15}$$

where variables with a tilde ($\tilde{}$) are in real terms (divided by $P_{H,t}^*$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of government debt.

In the transfer scenario, fiscal policy in country F chooses government consumption to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{2.6.16}$$

while using real transfers $\tilde{T}_t^*$ to deleverage its government debt, through the debt rule:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \tag{2.6.17}$$

and varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) Y_t^* - (\tau^{*s} + \tau^{*w} MC^*) Y^* = G_t^* - G^* \tag{2.6.18}$$

where $\rho_g^* \in [0,1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and ε_t is a zero mean white noise process. Variables without subscripts t represent their respective steady state level, while $\psi_y^* \geq 0$ represents the responsiveness of government consumption to variations of the output gap, $\gamma_t^* \in [0,1]$ is the desired share of reduction per period of the excess real government debt with respect to steady state. If γ_t^* increases over time then the deleveraging is backloaded, while if γ_t^* decreases over time then the deleveraging is frontloaded. If $\gamma_t^* = \gamma^*$ then the deleveraging is linear and constant in the desired fraction of reduction per period of the excess real government debt, as the fraction of deleveraging does not vary over time.

In the consumption scenario, fiscal policy in country F chooses real transfers to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} \tag{2.6.19}$$

while using government consumption G_t^* to deleverage its government debt, through the debt rule:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \tag{2.6.20}$$

and varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) Y_t^* - (\tau^{*s} + \tau^{*w} MC^*) Y^* = \tilde{T}_t^* - \tilde{T}^* \tag{2.6.21}$$

where $\rho_t^* \in [0,1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while $\psi_y^* \geq 0$ represents the responsiveness of real transfers to variations of the output gap.

In the distortionary tax scenario, fiscal policy in country F chooses government consumption to stabilize the output gap countercyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \tag{2.6.22}$$

while keeping real transfers constant and varying equally the tax rates on labour income and firm sales to deleverage its government debt, through the debt rule:

$$\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left(\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G} \right) \qquad \tilde{T}^*_t = \tilde{T}^* \tag{2.6.23}$$

and also to finance the remaining government expenditure, through the tax rule:

$$\tau^{*w}_t - \tau^{*w} = \tau^{*s}_t - \tau^{*s} \tag{2.6.24}$$

2.7 Government and Fiscal Policy in a Coordinated Currency Union

If the Governments of the two countries choose to coordinate, they will use their fiscal instruments to target a common objective, while maintaining independent budget constraints. In a Coordinated Currency Union scenario, instead of using government consumption or transfers to stabilize the domestic output gap countercyiclically, they will use the same fiscal instruments to stabilize the net exports gap procyclically, while using the other to deleverage government debt and using a mix of distortionary tax rates to finance remaining fiscal policy or to finance also deleveraging. This represents the act of coordinating policies on a common objective, which depends on the interactions between the two economies. The budget constraints of the two fiscal authorities instead remain unmodified. Here Germany balances its budget and the Eurozone Periphery deleverages its government debt, still managing fiscal policy independently, but coordinating by stabilizing their trade flows.

In the transfer scenario, fiscal policy in country H chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G} \right)^{\rho_g} e^{\varepsilon_t} \tag{2.7.1}$$

while keeping real transfers constant and maintaining a balanced budget by varying equally the tax rates, as in the Pure Currency Union scenario.

In the same transfer scenario, fiscal policy in country F chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G^*_t}{G^*} = \left(\frac{\widetilde{NX}^*_t}{\widetilde{NX}^*} \right)^{\psi^*_{nx}(1-\rho^*_g)} \left(\frac{G^*_{t-1}}{G^*} \right)^{\rho^*_g} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}} \right)^{-\psi^*_{nx}(1-\rho^*_g)} \left(\frac{G^*_{t-1}}{G^*} \right)^{\rho^*_g} e^{\varepsilon_t} \tag{2.7.2}$$

while using real transfers $\tilde{T}^*_t$ to deleverage its government debt and varying equally the tax rates, as in the Pure Currency Union scenario.

In the consumption scenario, fiscal policy in country H chooses real transfers to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{2.7.3}$$

while keeping government consumption constant and maintaining a balanced budget by varying equally the tax rates, as in the Pure Currency Union scenario.

In the same consumption scenario, fiscal policy in country F chooses real transfers to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}^*_t}{\tilde{T}^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi^*_{nx}(1-\rho^*_t)} \left(\frac{\tilde{T}^*_{t-1}}{\tilde{T}^*}\right)^{\rho^*_t} e^{\varepsilon_t} \tag{2.7.4}$$

while using government consumption G^*_t to deleverage its government debt and varying equally the tax rates, as in the Pure Currency Union scenario.

In the distortionary tax scenario, fiscal policy in country H chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{2.7.5}$$

while keeping real transfers constant and maintaining a balanced budget by varying equally the tax rates, as in the Pure Currency Union scenario.

In the same distortionary tax scenario, fiscal policy in country F chooses government consumption to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G^*_t}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi^*_{nx}(1-\rho^*_g)} \left(\frac{G^*_{t-1}}{G^*}\right)^{\rho^*_g} e^{\varepsilon_t} \tag{2.7.6}$$

while keeping real transfers constant and varying equally the tax rates, as in the Pure Currency Union scenario, and where $\psi_{nx} \geq 0$ for country H and $\psi^*_{nx} \geq 0$ for country F represent the responsiveness of either government consumption or real transfers to variations of the real net exports gap.

2.8 Government and Fiscal Policy in a Full Fiscal Union

If instead of considering two fiscal authorities managing fiscal policy independently, one for each country, or coordinating their policies, but with two separate budget constraints, we consider only one fiscal authority managing fiscal policy for both countries at the same time in a coordinated way and with a consolidated budget constraint, then we can think of it as an extreme case of fiscal policy coordination. A Full Fiscal Union scenario uses a consolidated budget constraint to finance local government consumption or transfers for international stabilization purposes, by setting them to target the net exports gap, while using the other to deleverage government debt and using a mix of distortionary tax rates to finance remaining fiscal policy or to finance also deleveraging, but varying equally the fiscal instruments across countries, so as to use union-wide resources to finance the government expenditure and the deleveraging. In this case government debt will be aggregated across countries and both countries will contribute to the deleveraging of government debt. Nonetheless, given that financial markets are still incomplete, there continue to be two separate government bonds for the two countries, which pay different interest rates and so have different bond yields. Here Germany and the Eurozone Periphery do not manage fiscal policy independently anymore and, while coordinating by stabilizing their trade flows, they also harmonize the movements of their fiscal instruments to finance both countries expenditures and to deleverage the government debt of the Eurozone Periphery, as if there were only one country. Germany in this case pays partially for the deleveraging of the Eurozone Periphery.

A Full Fiscal Union uses local government spending to manage fiscal policy at the union level with a consolidated budget constraint. The Fiscal Union finances streams of local public consumption, G_t and G_t^*, and transfers, T_t and T_t^*, subject to the consolidated budget constraint of the two national fiscal authorities:

$$P_{H,t}G_t + P_{H,t}^*G_t^* + T_t + T_t^* + B_{t-1}^G(1+i_{t-1}) + B_{t-1}^{*G}\frac{1+i_{t-1}}{1-\delta_{t-1}} = \\ B_t^G + B_t^{*G} + \tau_t^s P_{H,t}Y_t + \tau_t^{*s}P_{H,t}^*Y_t^* + \tau_t^w W_t N_t + \tau_t^{*w}W_t^*N_t^* \quad (2.8.1)$$

Dividing the government budget constraint by $P_{H,t}$ yields the government budget constraint in real terms (for country H):

$$G_t + \tilde{T}_t + S_t(G_t^* + \tilde{T}_t^*) + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}\frac{S_{t-1}\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \\ (\tau_t^s + \tau_t^w MC_t d_t)Y_t + (\tau_t^{*s} + \tau_t^{*w}MC_t^* d_t^*)S_tY_t^* + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + S_t\tilde{B}_t^{*G} - \frac{S_{t-1}\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (2.8.2)$$

where variables with a tilde (˜) are in real terms (divided either by $P_{H,t}$ or by $P_{H,t}^*$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of overall government debt.

In the transfer scenario, union-wide fiscal policy chooses government consumption in each country to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)}\left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \quad (2.8.3)$$

$$\frac{G_t^*}{G^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*}\right)^{\psi_{nx}^*(1-\rho_g^*)}\left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)}\left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \quad (2.8.4)$$

while using real transfers equally in both countries to deleverage the government debt of country F, while country H maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left(\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G} \right) \qquad \tilde{B}^G_t = \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} \qquad \tilde{T}_t - \tilde{T} = \tilde{T}^*_t - \tilde{T}^* \tag{2.8.5}$$

while varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau^w_t - \tau^w = \tau^s_t - \tau^s \qquad \tau^{*w}_t - \tau^{*w} = \tau^w_t - \tau^w \qquad \tau^{*s}_t - \tau^{*s} = \tau^s_t - \tau^s \tag{2.8.6}$$

$$(\tau^s_t + \tau^w_t MC_t d_t) Y_t + (\tau^{*s}_t + \tau^{*w}_t MC^*_t d^*_t) S_t Y^*_t - (\tau^s + \tau^w MC) Y - (\tau^{*s} + \tau^{*w} MC^*) Y^* = G_t + G^*_t - G - G^* \tag{2.8.7}$$

In the consumption scenario, union-wide fiscal policy chooses real transfers in each country to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}} \right)^{\rho_t} e^{\varepsilon_t} \tag{2.8.8}$$

$$\frac{\tilde{T}^*_t}{\tilde{T}^*} = \left(\frac{\widetilde{NX}^*_t}{\widetilde{NX}^*} \right)^{\psi^*_{nx}(1-\rho^*_t)} \left(\frac{\tilde{T}^*_{t-1}}{\tilde{T}^*} \right)^{\rho^*_t} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}} \right)^{-\psi^*_{nx}(1-\rho^*_t)} \left(\frac{\tilde{T}^*_{t-1}}{\tilde{T}^*} \right)^{\rho^*_t} e^{\varepsilon_t} \tag{2.8.9}$$

while using government consumption equally in both countries to deleverage the government debt of country F, while country H maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G}_t = \gamma^*_t \left(\frac{\tilde{B}^{*G}_{t-1}}{\Pi^*_{H,t}} - \tilde{B}^{*G} \right) \qquad \tilde{B}^G_t = \frac{\tilde{B}^G_{t-1}}{\Pi_{H,t}} \qquad G_t - G = G^*_t - G^* \tag{2.8.10}$$

while varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau^w_t - \tau^w = \tau^s_t - \tau^s \qquad \tau^{*w}_t - \tau^{*w} = \tau^w_t - \tau^w \qquad \tau^{*s}_t - \tau^{*s} = \tau^s_t - \tau^s \tag{2.8.11}$$

$$(\tau^s_t + \tau^w_t MC_t d_t) Y_t + (\tau^{*s}_t + \tau^{*w}_t MC^*_t d^*_t) S_t Y^*_t - (\tau^s + \tau^w MC) Y - (\tau^{*s} + \tau^{*w} MC^*) Y^* = T_t + T^*_t - T - T^* \tag{2.8.12}$$

In the distortionary tax scenario, union-wide fiscal policy chooses government consumption in each country to stabilize its real net exports gap procyclically, while following in part an exogenous process, through the fiscal rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G} \right)^{\rho_g} e^{\varepsilon_t} \tag{2.8.13}$$

$$\frac{G_t^*}{G^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*}\right)^{\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{2.8.14}$$

while keeping real transfers constant and varying equally across countries the tax rates on labour income and firm sales to deleverage the government debt of country F, while country H maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \tilde{T}_t = \tilde{T} \qquad \tilde{T}_t^* = \tilde{T}^* \tag{2.8.15}$$

while also varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^w - \tau^w \qquad \tau_t^{*s} - \tau^{*s} = \tau_t^s - \tau^s \tag{2.8.16}$$

3 Calibration

The model is calibrated[4] following our previous work, Cole, Guerello and Traficante (2016), which mainly follows Ferrero (2009), so we consider the top 5 Eurozone countries, which account for more than 80% of Eurozone GDP and we divide them into the Periphery (namely, France, Italy, Spain and The Netherlands), country F, and the Core (namely Germany), country H. The size of country H is set according to the relative GDP size to $h = 0.4$, as Germany accounts for over 35% of Eurozone GDP.

As in Ferrero (2009) most of the parameters governing the economies of the two countries are set symmetrically, with the exception of the degree of price rigidity, which has been set such that in country H the average duration of a price is 4 quarters while in country F it is 5 quarters. The gross markup $\frac{\varepsilon}{\varepsilon-1}$ has been set to 1.1, which implies a net markup of 10%, and the discount factor has been chosen to match a compounded annual interest rate of 2%. The parameters for monetary policy follow common values used in the literature, so we set the response of the interest rate to inflation to $\phi_\pi = 1.5$, according to the Taylor principle, and the interest rate smoothing parameter to $\rho_i = 0.8$. We estimate the sensitivity of the transaction cost, or of the government bond spread, δ_t to deviations of government debt-to-GDP from steady state and find that for every ten percentage points increase in government debt-to-GDP the government bond spread increases by 9 percentage points[5], according to which we set $\delta^B = 0.009$. Table 1 collects all calibrated parameters and steady state stances.

The calibration of the two countries mainly differs in the fiscal policy parameters. In particular, the government consumption-to-GDP ratios have been set respectively to 18.7% for Germany and 21.9% for the Periphery, according to the average of the last 9 years (source ECB-SDW). The marginal tax rates on labour income have been set respectively to 40.61% for Germany and

[4]The calibration is done on the steady state values described in Appendix A.2.

[5]This result is in line with Hjortsø (2016), which finds a similar sensitivity to be 0.01 instead of 0.009.

Table 1: Calibrated Parameters and Steady State Stances.

Parameters	Description	Country H	Country F
h	Relative size of domestic economy	0.4	0.6
β	Discount factor	0.995	0.995
ε	Elasticity of substitution of domestic goods	11	11
$\frac{\varepsilon}{\varepsilon-1}$	Gross Price Mark-Up	1.1	1.1
η	Elasticity of substitution foreign and domestic goods	4.5	4.5
σ	Inverse Elasticity of intertemporal substitution	3	3
φ	Inverse Frisch Elasticity of labour supply	0.5	0.5
θ	Degree of price rigidity	3/4	4/5
δ^B	Sensitivity of bond spread to debt-to-GDP deviations	0.009	0.009
γ	Desired reduction of excess government debt-to-GDP	0	0.05
α	Openness of domestic economy	0.52	0.361
$\frac{\alpha}{h}$	Relative openness of domestic economy	1.3	0.6017
$\frac{1-\alpha}{h}$	Home bias	1.2	1.065
ψ_y	Responsiveness of fiscal policy to output gap	0.067	0.061
ψ_{nx}	Responsiveness of fiscal policy to net exports gap	0.043	0.014
ϕ_π	Responsiveness of monetary policy to inflation	1.5	1.5
ρ_i	Interest Rate smoothing parameter	0.8	0.8
ρ_ξ	Persistence of preference shock	0.94	0.8
ρ_a	Persistence of technology shock	0.58	0.70
ρ_δ	Persistence of spread shock	0.95	0.95
σ_ξ	Standard deviation preference shock	0.0024	0.0086
σ_a	Standard deviation technology shock	0.0087	0.0033
$corr_\xi$	Correlation preference shock	0.625	0.625
$corr_a$	Correlation technology shock	0.418	0.418
Steady State Ratios	**Description**	**Country H**	**Country F**
$(1+i)^4-1$	Annualized Interest Rate	2%	2%
τ^w	Tax Rate on wage income	40.6%	27.9%
τ^s	Tax Rate on firm sales	2.5%	19.5%
$\tau^w MC + \tau^s$	Tax Revenues-to-GDP	38.49%	39.92%
$\frac{G}{Y}$	Government consumption-to-GDP	18.7%	21.9%
$\frac{\tilde{T}}{Y}$	Real Transfers-to-GDP	18.58%	16.81%
$\frac{\tilde{NX}}{Y}$	Net Exports-to-GDP	1.72%	-1.14%
$\frac{C}{Y}$	Consumption-to-GDP	79.58%	79.24%
$\alpha^* \frac{C^*}{Y}$	Exports-to-GDP	43.1%	27.47%

27.94% for the Periphery in accordance to the average in the last 9 years of the labour income tax wedges, excluding social security contributions made by the employer, for the median individual, as reported in OECD (2015). The marginal tax rate on firm sales has been set to 19.5% for the Periphery according to the average in the last 9 years of the VAT for France, Italy, Spain and The Netherlands as reported in Eurostat, European-Commission et al. (2015), while it has been calibrated for Germany to match the average ratio of net exports-to-GDP of 1.73% observed over the past 9 years for Germany[6]. Although the observed VAT rate for Germany is 19%, we set its marginal tax rate on firm sales to 2.5%, as if there was a production incentive, to correct for the fact that Germany has a greater productivity than the Periphery countries. This calibration implies a steady state tax revenue-to-GDP ratio of respectively 38.49% for Germany and 39.92% for the Periphery, clearly in line with the data observed over the past decades for Germany (38.72%) and for France, Italy, Spain and The Netherlands (39.15%).

Finally, the annualized steady state value of government debt-to-GDP in both countries is set to roughly 60% as stated in the Maastricht Treaty. In the simulations, the Periphery (country F) starts with a higher level of government debt-to-GDP, equal to roughly 80%, in line with the average level of government debt-to-GDP for France, Italy, Spain and The Netherlands. The desired fraction of reduction of excess government debt for the Periphery is set for most simulations to $\gamma = 0.05$ (with the same average for all other simulations), corresponding to a 5% yearly reduction of excess government debt[7], to comply with the Debt Brake Rule in the Fiscal Compact. The calibration of the fiscal policy parameters, ψ_y and ψ_{nx}, follows the optimizations done in our previous work, Cole, Guerello and Traficante (2016).

Since the two countries' fiscal policy ratios have been calibrated according to the data, the transfers-to-GDP ratios have been set such that the government deficit is zero in steady state. Henceforth, the overall calibration of the fiscal sector implies a steady state ratio of transfers-to-GDP of respectively 18.58% for country H and 16.81% for country F, and a steady state ratio of current expenditure-to-GDP of respectively 37.28% for country H and 38.71% for country F. This calibration is broadly in line with the observed data over the last 10 years for the subsidies-to-GDP ratio (26.85% for Germany and 24.69% for the Periphery countries) and the current expenditure (less interest)-to-GDP ratio (35.54% for Germany and 36.85% for the Periphery countries).

The parameters of openness have been set to match an export-to-GDP ratio $\left(\frac{\alpha^* C^*}{Y}\right)$ of roughly 43% for Germany[8] taken from the aggregate demand equation, while for the Periphery the parameter of openness is recovered by equating per-capita consumption across countries, which yields the

[6] The average current account to GDP ratio observed over the past 9 years for Germany is roughly 6.36%. However, we adjust the data for the overall trade weight with France, Italy, Spain and The Netherlands (26%).

[7] This corresponds to a similar reduction in government debt-to-steady state GDP.

[8] The value recovered from the data as the average of the last 9 years is 43.5%.

following equation:

$$\alpha^* = \frac{h}{1-h}\left[\alpha + \frac{\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\right)\left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})}\right)^{\frac{1}{\varphi}} - 1}{1+\frac{h}{1-h}\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}}\right)\left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})}\right)^{\frac{1}{\varphi}}}\right] \tag{3.0.1}$$

Consequently, home biases are given by $\frac{1-\alpha}{h} = 1.2$ and $\frac{1-\alpha^*}{1-h} = 1.065$. Since both home biases are larger than one it means that the share of consumption of domestic goods is higher than the share of production of domestic goods. Based on our calibration, the real exchange rate increases as the terms of trade increase because the degree of openness of country H ($\alpha = 0.52$) is less than the size of country F ($1 - h = 0.6$)[9].

Regarding the dynamic parametrization of the shocks, both technology and preference shocks are assumed to follow a VAR(1) process that generally allows for both direct spillovers and second order correlation of the innovations. However, the structure has been restricted for all shocks to exclude direct spillovers.

With the exception of the preference shocks, whose dynamics have been calibrated following Kollmann et al. (2014), the parameters characterizing the dynamics of the technology shocks have been estimated. For the estimation we have employed the time series for Germany, France, Italy and Spain of labour productivity per hours worked for the technology shocks. All the series are chain-linked volumes re-based respectively in 2010, seasonally adjusted and filtered by means of a Hodrick-Prescott filter. The sample considered spans at quarterly frequency from 2002 Q1 to 2015 Q3. The correlation between preference shocks follows our previous work, Cole, Guerello and Traficante (2016), where we pick the value that maximizes the simulated correlation between output in the two countries.

The transaction cost δ_t responds to the debt-to-GDP ratio in deviation from Maastricht Treaty's objective of 60%. It is assumed to follow an autoregressive AR(1) process. The parameters defining the AR(1) process and the response to the debt-to-GDP ratio are calibrated looking at the data for the spread between the long run interest rates on government bonds[10] for France, Italy, Spain and The Netherlands and the same yield for Germany. These series are combined with data on the government debt-to-GDP ratio (as a difference from Maastricht Treaty's objective) using a panel VAR technique and taking the estimated parameters in the bond spread equation and the estimated variance of the residuals. In order to account for the effects of the European sovereign debt crisis, the data on the spread has been demeaned and country fixed effects have been introduced in the model to account for the initial conditions (i.e. few countries show historically high levels of government debt).

[9]See Cole, Guerello and Traficante (2016) for details.

[10]The data is collected by Eurostat and reported in the ECB statistical data warehouse. It is harmonized to assess the convergence of the Member Countries. The sample spans between 2002 and 2015 and features quarterly frequency.

4 Numerical Simulations

We simulate the model numerically using Dynare[11] (Adjemian et al., 2011), which takes a first-order approximation[12] of the model around its symmetric non-stochastic steady state with zero inflation and constant government debt. We compare the Impulse Response Functions of the main variables to negative shocks of one standard deviation of different nature and to deleveraging by country F from 80% government debt-to-GDP to its long-run value of 60% government debt-to-GDP, under a range of fiscal policy specifications, to study the stabilization properties of different fiscal policy instruments for deleveraging, different fiscal policy coordination strategies, and different deleveraging schemes, and to study the international transmission of shocks with incomplete financial markets, also at the Zero Lower Bound.

4.1 Deleveraging Schemes

Country F (the Periphery) features an initial level of government debt-to-GDP of about 80%, which is higher than the target of fiscal policy, namely a debt-to-GDP level of 60%. The deleveraging scheme implies that country F has to gradually reduce its government debt over time by using a fiscal instrument, either government consumption, transfers or distortionary taxes. In the baseline calibration, Country F has to reduce its government debt by 5% of the excess each quarter until it goes back to its long-run target, as stated in the Maastricht Treaty.

To identify the effects of a prolonged period of austerity and deleveraging in an economy, we analyze the transmission mechanism of a permanent shock to the government debt target, which brings it from a long-term value of 80% to 60%. This implies that the fiscal authority after the shock must adjust taxes and, eventually, government transfers to balance the budget and to reduce the debt-to-GDP ratio. Furthermore, to fully understand the effects of the deleveraging scheme, it is interesting to analyze how a different speed of government debt reduction affects the transmission of the shock to the government debt target. Specifically, we assume the coefficient governing the deleveraging rule, γ_t^*, to be time-varying and its process to be determined by the following alternative paths:

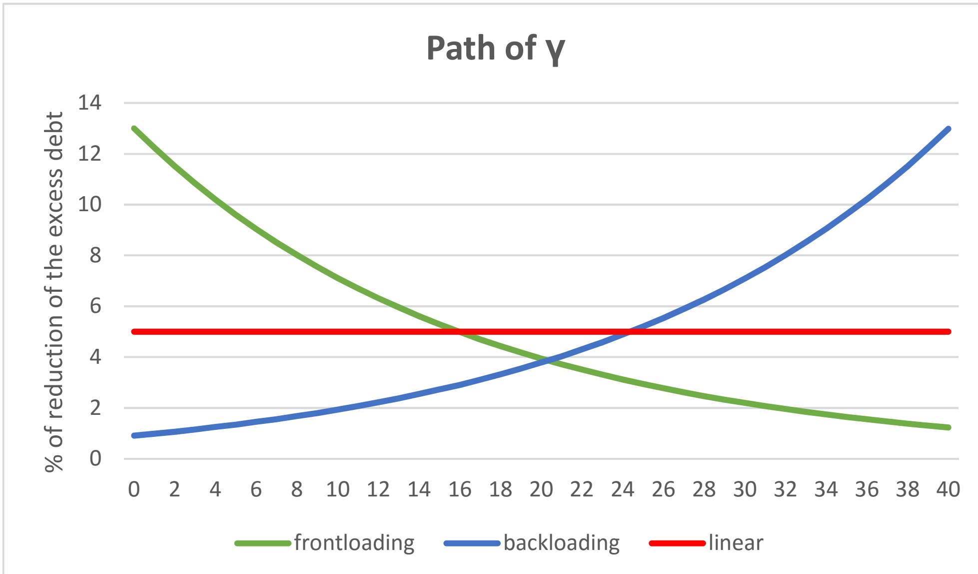
- Frontloading: the amount and consequent cost of deleveraging is higher initially and decreases over time as the level of excess government debt goes down. This is achieved through a path for γ_t^* that starts from a level of roughly 13% and is reduced to 0.1% in 10 years (40 quarters). However, for this path to be comparable with the baseline case, it is designed such that the average reduction of excess government debt is 5%, as in the baseline calibration.

- Backloading: the amount and consequent cost of deleveraging is more evenly distributed over time, as the percentage of desired reduction of excess government debt increases over time, while the excess government debt decreases. The features of this path are symmetric to the

[11] All the equilibrium conditions of the model used for the simulations are shown in Appendix A.1.

[12] All simulations are given by first-order approximations of the equilibrium conditions, except for Section 4.1, where the simulations are given by second-order approximations of the equilibrium conditions.

Figure 1: Paths for γ_t^* with different Deleveraging Schemes



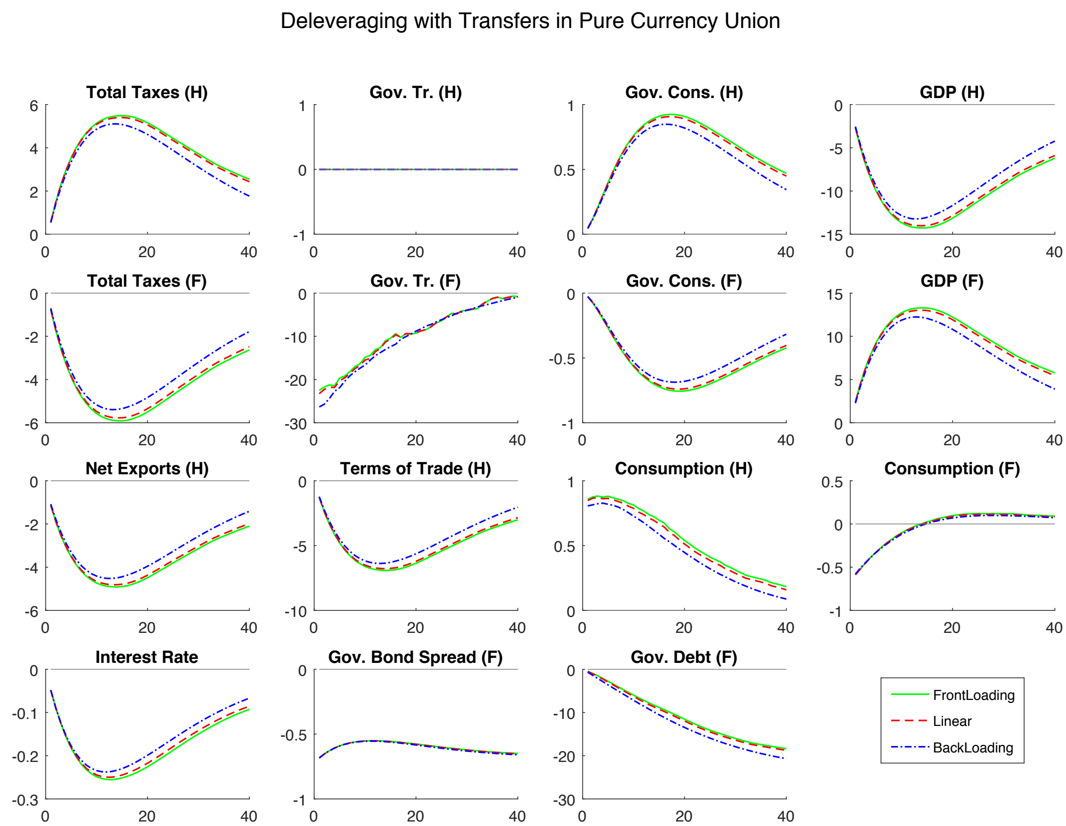
one assumed for the frontloading case. Specifically, γ_t^* starts from a level of roughly 1% and increases to 10% in 10 years (40 quarters), while the average value for γ_t^* is roughly 5%.

The three paths for γ_t^* are shown in Figure 1. We can notice that the frontloading case implies a higher cost of deleveraging than the baseline case in the first 4 years and the backloading case implies a lower speed of deleveraging for roughly 5 years.

Figure 2 compares different deleveraging schemes in the Pure Currency Union scenario, where transfers are used to deleverage the government debt of country F, while taxes balance the budget. The shock to the government debt target in country F implies that from the first quarter onwards government transfers are not kept constant any longer, but they adjust to reduce the government debt by the desired amount. Therefore, the cost of deleveraging affects negatively the economy through a wealth effect (transfers) on households in country F. Furthermore, since the government bond spread depends on the distance between the actual debt-to-GDP ratio and its long-run target, it falls following the deleveraging shock.

Looking at Figure 2, we can observe that government transfers strongly decrease by almost 30% on impact and persist below their steady state for a very long period, because the fiscal authority finances the deleveraging by cutting its expenditure. The first effect is that also private consumption in country F decreases on impact, but since the wealth effect on consumption is partially balanced

Figure 2: Deleveraging Schemes - Government Transfers - Pure Currency Union



by the response of the bond spread to a decreasing debt-to-GDP ratio, consumption reverts back to its long run level in less than 5 years. This mechanism is also amplified by the strong response of monetary policy to the deflationary pressure induced by the deleveraging process. The decrease in the interest rate set by the central bank further decreases the interest rate in country F ($i_t^* \approx i_t + \delta_t$), because both its components, after the initial adjustment to the new target, decrease over time following both fiscal and monetary policy responses to the shock. Furthermore, the lower interest rate in country H brings households to increase current consumption, generating international spillovers, which however benefit country F's economy. It is possible to notice that the terms of trade in country H and thus net exports fall after the shock due to the lower price rigidity in country H. The consequent drop in GDP in country H and increase in country F partially mitigates the cost of deleveraging. The effect of an increase in GDP in country F is twofold: on one hand it further decreases the debt-to-GDP ratio and hence it stabilizes the bond spread; on the other hand it increases the tax base, pushing the fiscal authority to reduce the tax rates to balance the government budget.

Comparing the different deleveraging schemes, we notice that as long as the wealth effect given by the decrease in government transfers dominates other effects, as shown by private consumption remaining below its long run level for roughly the first 12 quarters, the difference in the speed of deleveraging can hardly be identified. The reduction in real government debt implies a deflationary pressure on the economy, that is reduced after about 4 years, as the excess government debt is cut. It is possible to observe in Figure 2 that after 12 quarters the paths of the interest rate and consumption in country H differ across the deleveraging schemes, while the bond spread and consumption in country F do not, as they are less affected by inflation. Specifically, we can see that with the backloading scheme, due to a higher implied deleveraging rate after 5 years, deflationary pressures reduce faster, implying more stabilization in the economy. This effect is asymmetric over time, because the higher deleveraging rate implied by the frontloading scheme in the first 5 years follows almost the same path of the linear deleveraging scheme, rather than being more amplified.

In summary, this analysis shows that the speed of deleveraging in the Pure Currency Union scenario matters only in the medium-to-long run, when most variables start to converge back to steady state, because in the short run both economy's dynamics are largely driven by movements in the bond spread and in government transfers. Therefore the backloading scheme is more stabilizing than both the linear and frontloading schemes in all scenarios, because it affects the dynamics later in time when the the wealth effect driven by government transfers starts to vanish.

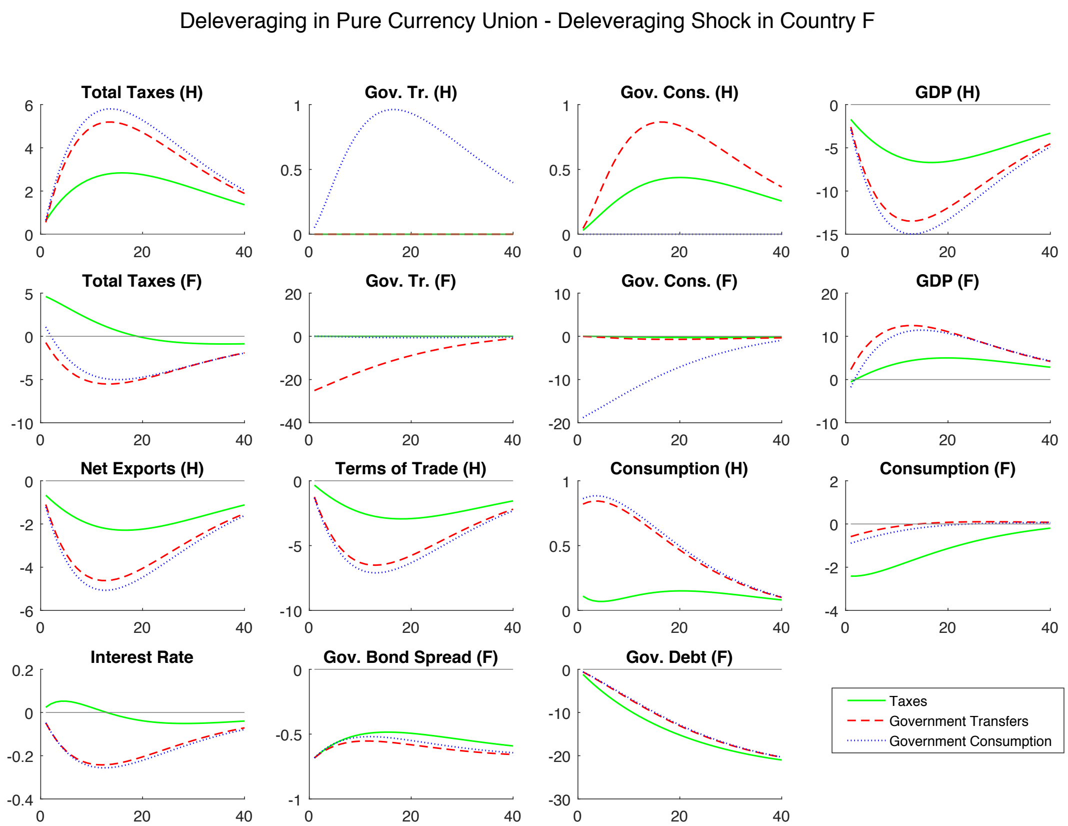
4.2 Instruments for Deleveraging

Figures 3 and 4 show the comparison between different fiscal instruments for deleveraging after a shock to the government debt target in country F, which brings it from 80% to 60% of GDP before deleveraging, in the Pure Currency Union and in the Full Fiscal Union scenario, respectively. The increase in distortionary taxes rather than the decrease in government consumption or transfers to reduce government debt yield very different dynamics, changing spillovers across countries, which is why we study the stabilization properties of the different instruments.

In Figure 3 we see the dynamics of Government Consumption and Transfers in country F only when they are used to deleverage government debt because the movements of the instruments for deleveraging are of one order of magnitude greater than the movements of the instruments for stabilization. Moreover we can see the great volatility of GDP, which is given by the strong deflationary pressure produced by the deleveraging shock (as shown by the movements in the interest rate), and is reinforced by the relative movements in the tax rates, which make prices in country F decrease more than in country H, as shown by the movements in the terms of trade.

Strangely enough, after a deleveraging shock to country F, it is country H which falls into a recession, while country F has a boom. This effect is given by the greater openness to trade of country H, which drives the negative spillover from country F to country H, through an increase in the share of consumption of foreign goods which are more convenient because of the fall in the terms of trade. This boosts labour supply and thus GDP in country F. Country H must also increase its private consumption because the deleveraging of country F reduces available assets for savings

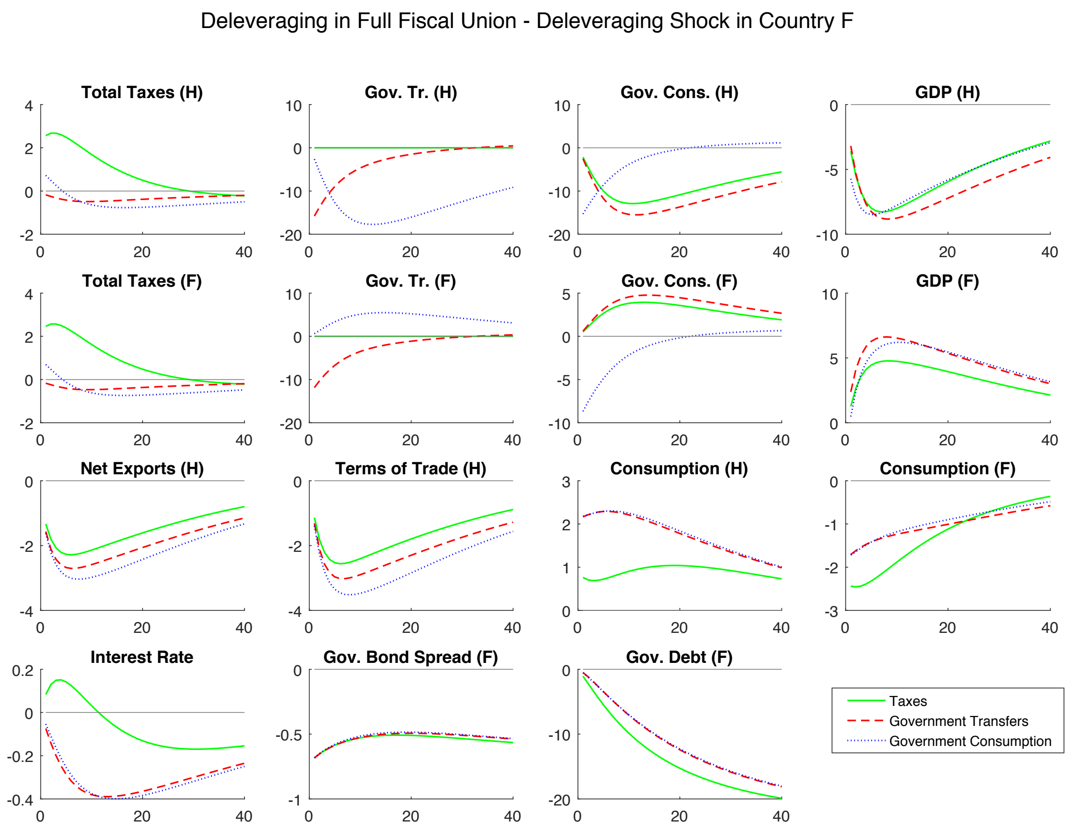
Figure 3: Instruments for Deleveraging - Pure Currency Union - Deleveraging Shock in Country F



mainly for households in country H, which hold most of the public debt of country F, other than because of the fall in the interest rate which boosts consumption.

Looking at Figure 3 we can see that there are hardly any differences in the stabilization properties of government consumption and government transfers as instruments for deleveraging, while using distortionary taxes to deleverage government debt produces more stable dynamics for most variables, except for consumption in country F. At the same time distortionary taxes affect the dynamics of the economy more than other fiscal instruments, but depending on their effect on consumption and especially prices, they can reduce the deflationary pressure compared to other instruments for deleveraging, bringing in the end to a gain in stabilization. Figure 3 shows that most variables follow qualitatively very similar paths, except for the fiscal policy instruments in country F. The fiscal policy instruments behave differently by construction. Taxes move much more when government consumption or transfers are used to deleverage, because they need to counteract the effect of the larger change in GDP on the government budget constraint. Instead, when deleveraging with taxes, taxes change sign in country F and increase instead of decrease, reducing

Figure 4: Instruments for Deleveraging - Full Fiscal Union - Deleveraging Shock in Country F



consumption in country F and reducing the volatility in the terms of trade, which stabilizes more net exports and GDP in both countries.

Figure 4 shows the same deleveraging dynamics, but in the Full Fiscal Union scenario. The first thing that can be noticed is that there is less divergence for most variables in the paths given by the different instruments for deleveraging, compared to the Pure Currency Union scenario. This is mainly because the tax rates and the fiscal instruments for deleveraging move jointly across countries, which stabilizes more relative prices and thus international spillovers, reducing the volatility in most variables. Nonetheless, the ranking by stabilization property of the fiscal instruments for deleveraging does not change in the Full Fiscal Union scenario. As in the Pure Currency Union scenario, distortionary taxes provide more stabilization than other fiscal instruments for most variables, except for consumption in country F. In general, in the Full Fiscal Union scenario there is more deflationary pressure on the economy (as shown by the difference in the scale of the movements in the interest rate) with respect to the Pure Currency Union scenario, because the consolidation of budget constraints and the joint movement in the tax rates and in the deleveraging instruments put the burden of debt reduction and consequent deflation also on country H. Nonetheless, distortionary taxes are able to reduce the volatility in the economy especially by moving in the same direction across countries, thus reducing the movements in the terms of trade and consequently international spillovers.

To conclude, using distortionary taxes to deleverage, rather than government consumption or transfers, stabilizes the economy the most in all scenarios, because it reduces the deflationary pressure in the economy and international spillovers.

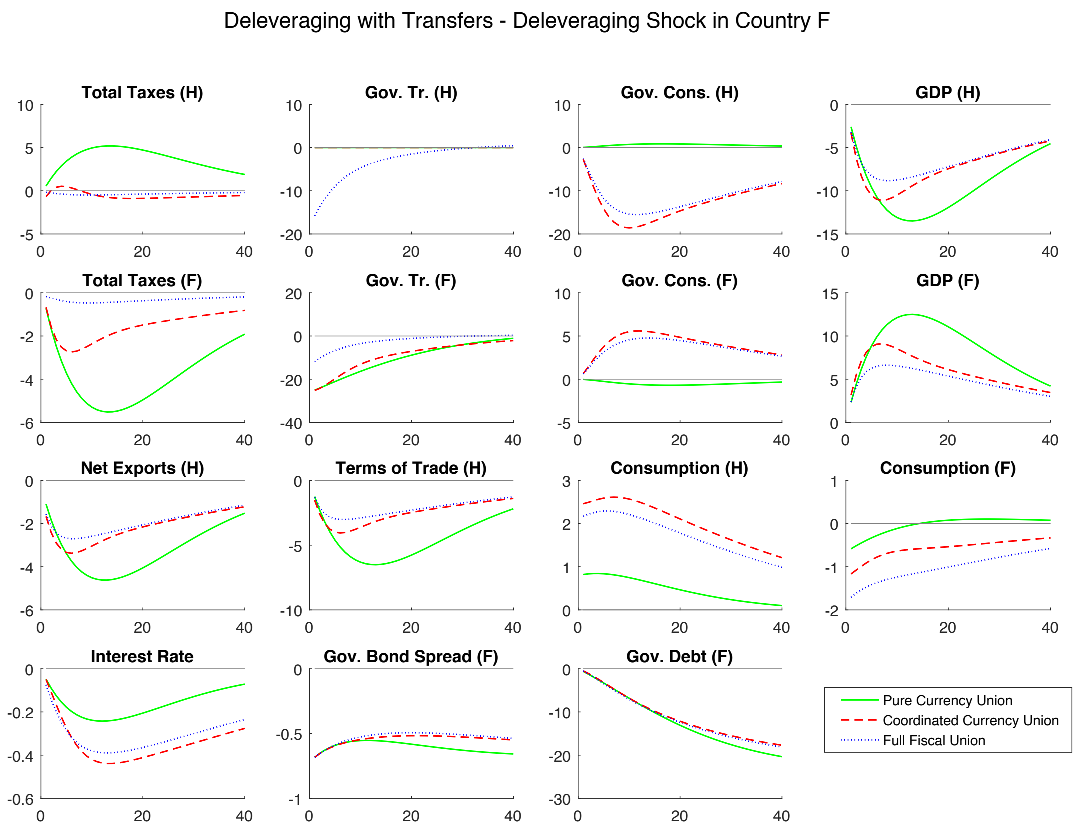
4.3 Coordination of Deleveraging

Figures 5 and 6 compare the three different degrees of fiscal policy coordination, or the three different scenarios, while using government transfers or taxes, respectively, to deleverage the government debt of country F. We can see in both cases how there is very little difference in the dynamics of all variables between the Full Fiscal Union scenario and the Coordinated Currency Union one. This tells us that coordinating fiscal policy, by targeting the net exports gap or by additionally consolidating budget constraints and moving tax rates jointly, makes little difference. In addition we can see that when using taxes to deleverage there is less difference in the dynamics of all variables between the three scenarios, making it difficult to see which scenario produces more stabilization. Only when using government transfers to deleverage we can see that the Pure Currency Union scenario is the worst in terms of stabilization of the deleveraging shock with incomplete markets, at least for GDP. This is because stabilizing the net exports gap (as in the other two scenarios) additionally reduces the volatility of distortionary taxes and consequently stabilizes relative prices and international spillovers.

Looking at Figure 5 we can see the stabilization properties of the different scenarios when deleveraging with government transfers. It is clear that the Full Fiscal Union scenario stabilizes more than the other scenarios the deleveraging shock, at least for GDP which is very volatile. The fact that both countries coordinate or share the cost of deleveraging reduces the amplification of the shocks, which is mainly given by the opposite movement of the tax rates in country H, which by falling instead of increasing reduce the movements in the terms of trade and consequently the negative spillovers driven by net exports. This creates a higher deflationary pressure, which can be seen by the more amplified movements in the interest rate. In the Full Fiscal Union scenario (and to a smaller extent in the Coordinated Currency Union scenario), since real transfers and taxes in country H move jointly with real transfers and taxes in country F, the lower taxes in country H make consumption in country H increase more and real government debt in country F decrease less, because of the effect of deflation, and thus GDP decreases less in country H. Overall, the Full Fiscal Union scenario is more stabilizing than the other two scenarios, especially for GDP and the terms of trade.

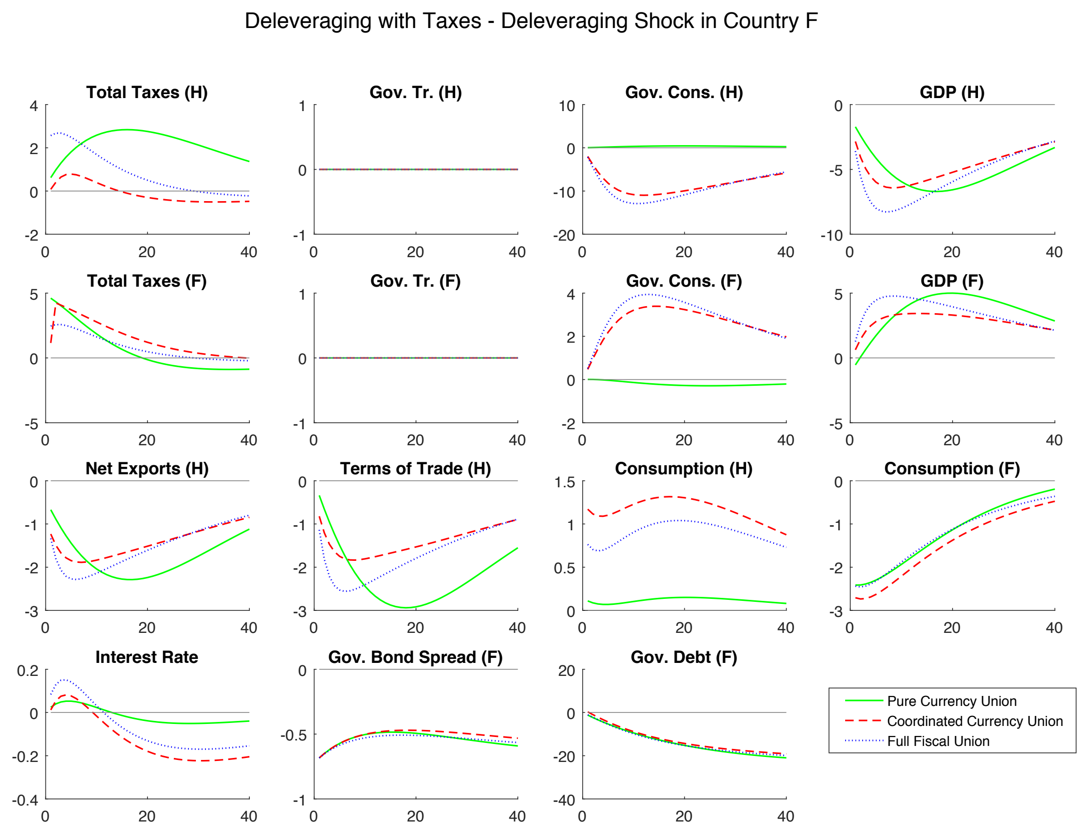
Looking at Figure 6, most of the reasoning done when using real transfers to deleverage still holds when using taxes. In this case taxes finance all expenditure and deleveraging and actually reduce the volatility of GDP and inflation compared to deleveraging with transfers, while at the

Figure 5: Coordination of Deleveraging - Government Transfers - Deleveraging Shock in Country F



same time reducing the difference in stabilization properties between the three scenarios. This shows how the main drivers of the dynamics are given by the variations in GDP in both countries, which are mainly given by the deflationary effect of the deleveraging shock on net exports through the terms of trade. When using taxes to deleverage, the Coordinated Currency Union scenario seems to produce more stabilization than the other two scenarios, especially for the terms of trade and net exports, while it is more difficult to see it for GDP. This brings us to think that it might not be convenient to consolidate budget constraints across countries in this case, as it reduces the stabilization gains from coordinating on the net exports gap. It does make it more difficult to compare scenarios for fiscal policy coordination if the stabilization properties depend on the fiscal instrument used to deleverage, but it seems that some sort of coordination does create more stabilization in the economy, although a welfare measure is needed to quantify it and make a precise comparison.

Figure 6: Coordination of Deleveraging - Taxes - Deleveraging Shock in Country F



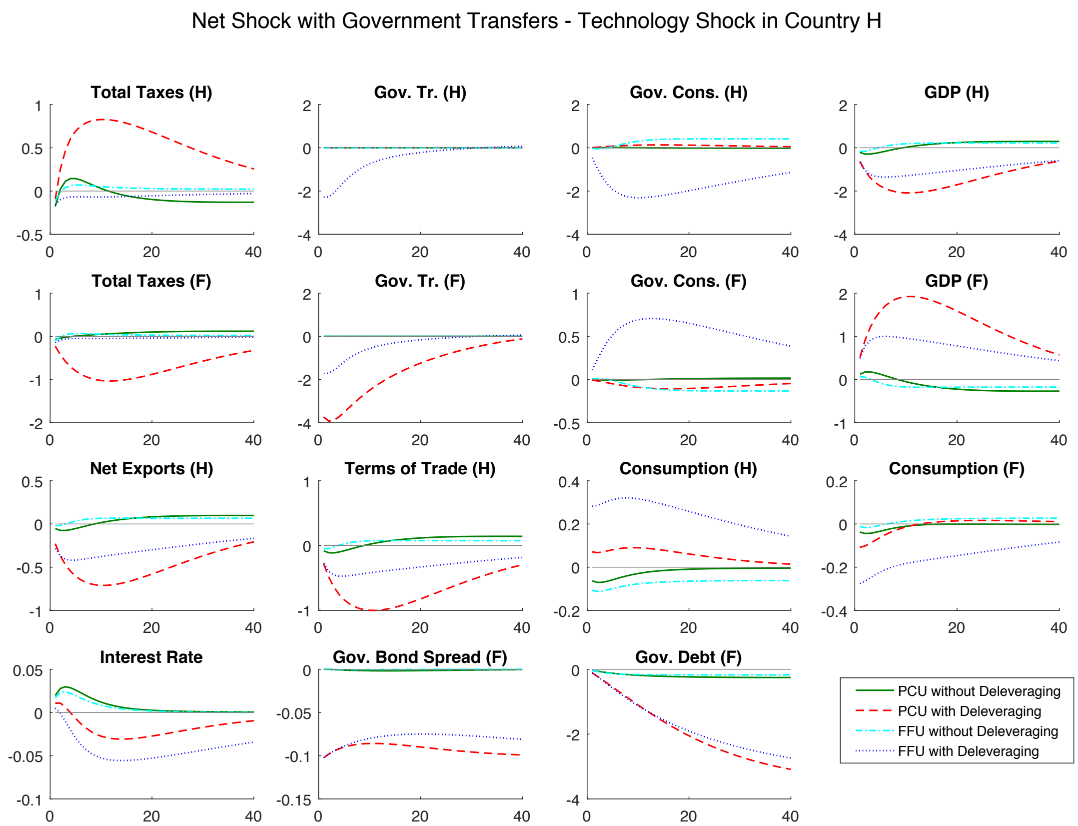
4.4 Net Shocks from Deleveraging

Here we analyze the effects of a technology shock in country H, showing the dynamics with and without deleveraging in country F[13]. The huge government debt deleveraging process that the fiscal authority is forced to follow makes the effects of other shocks marginal on the economy. For this reason, in this Section we try to isolate the effects of a productivity shock from those of the shock to the government debt target.

In Figure 7 we show the impulse response functions to a negative technology shock in country H, while keeping government debt constant (PCU and FFU without deleveraging) and while deleveraging (PCU and FFU with deleveraging). In the latter case, the impulse responses are presented in deviation from the path implied by the government debt reduction, by subtracting the effect of the pure deleveraging shock. This way we try to disentangle the effects of the shock on the government debt target from the constraint it imposes on fiscal policy stabilization, while keeping the effects of

[13]We also analyze the effects of a preference shock in country F, but we omit the simulation because the results are qualitatively similar.

Figure 7: Net Shock with Government Transfers - Technology Shock in Country H



the technology shock for all cases[14]. After a negative technology shock in country H, the increase in marginal costs for firms pushes prices up, while the terms of trade deteriorate, reducing net exports and output in country H on impact. The assumption of incomplete markets gives rise to two different interest rates for the households in the two countries, which in turn gives rise to the differences in consumption and asset allocation in the two countries, because of the deviation of government debt-to-GDP in country F from its steady state.

In Figure 7 we distinguish the case in which country F does not deleverage from the case in which the government in country F uses transfers to deleverage. The graph shows that deleveraging makes the dynamics more volatile in both the Pure Currency Union and Full Fiscal Union scenario. When country F deleverages, we observe a bigger contraction in the terms of trade and in net exports for country H, while CPI inflation decreases rather than increase, as can be inferred from the dynamic response of the nominal interest rate. The decrease in the interest rate is beneficial for

[14]A similar procedure is followed by Bodenstein, Guerrieri and Gust (2013) which presents impulse responses in deviation from the path implied by a severe recession.

consumption in country H, which moves in the opposite direction with respect to consumption in country F, also because of the assumption of incomplete markets. Notice that without deleveraging the divergence in household consumption in the two countries is greatly reduced and does not occur on impact. The interest rates and total taxes are the variables that determine how much consumption dynamics diverge between country H and country F. While without deleveraging the dynamics in both the Pure Currency Union and Full Fiscal Union scenario are quantitatively similar, when the government in country F has to deleverage, the Full Fiscal Union scenario stabilizes most variables of the economy more than the Pure Currency Union one. In particular, output in both countries is less volatile and international spillovers are reduced, because government transfers and taxes move jointly in both countries, reducing the volatility of real government debt in country F.

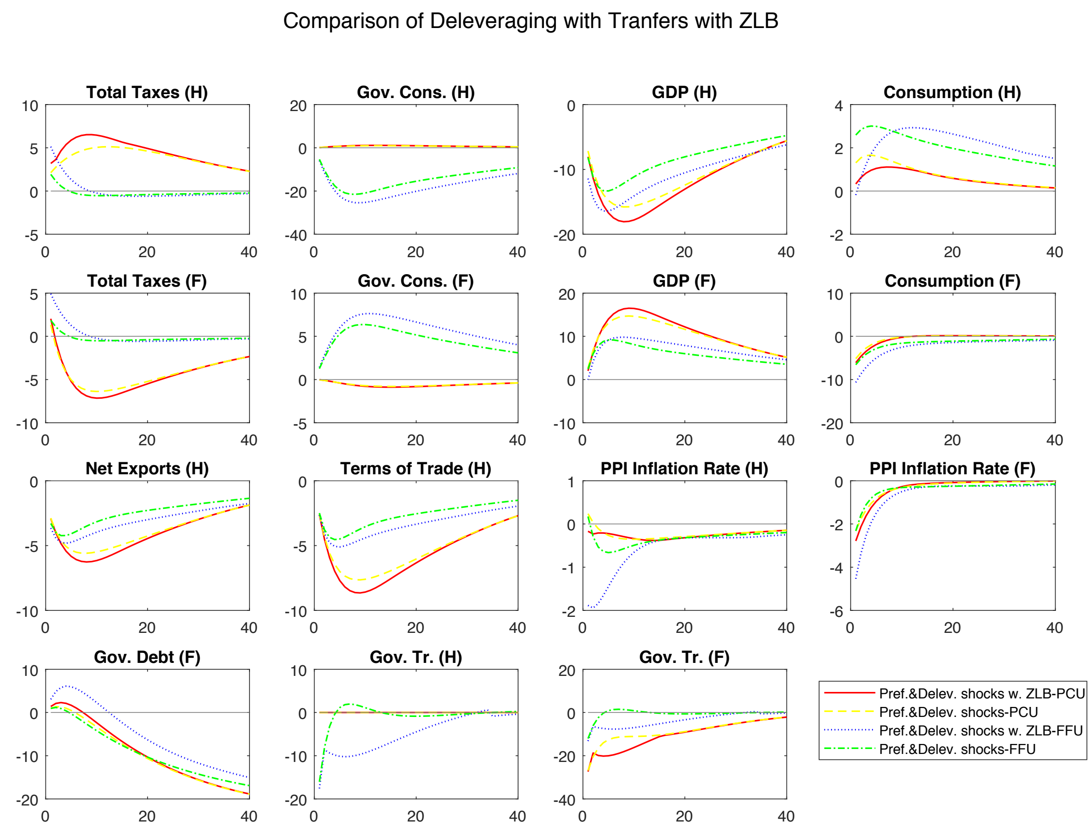
4.5 Coordination of Deleveraging at the Zero Lower Bound

It is well known that a monetary policy which aims at price stabilization typically offsets the stimulative effect of an expansionary fiscal policy. An expansionary fiscal action leads to an increase in inflationary pressure, which in turn leads to movements in the real interest rate. If the Central Bank is active, the inflationary pressures lead to upward movements in the real interest rate, while if the Central Bank is passive the real interest rate decreases, amplifying the effect of the fiscal stimulus. As argued by Coenen et al. (2012), although in Europe this amplification mechanism is less pronounced than in the US, due to greater nominal rigidities, it is quite relevant for the transmission of fiscal shocks. Furthermore, Erceg and Lindé (2014) points out to the fact that an economy emerges faster from a liquidity trap if government spending rises and in such a trap sizeable government spending can stimulate a much larger response in tax revenues than in normal times, making fiscal expansions less costly and, eventually, a free lunch. The flip-side of these arguments, which are relevant when deleveraging government debt, is that spending cuts have larger effects if the nominal interest rate is up against the zero lower bound, while they may also prolong the recession and boost government debt, creating a vicious debt-deflation loop. For these reasons, we decide to compare the dynamics of the economy in the Pure Currency Union scenario and the Full Fiscal Union one, after a deleveraging shock which may bring the nominal interest rate to hit the zero lower bound. In this case the monetary policy rule moves from Equation 2.5.1 to:

$$\begin{aligned} i_t &= \max\{\tilde{i}_t, 0\} \qquad (4.5.1) \\ \beta(1+\tilde{i}_t) &= \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} \left[\beta(1+\tilde{i}_{t-1})\right]^{\rho_i} \end{aligned}$$

where $\tilde{i}_t$ is the shadow interest rate, which is the unconstrained level of the nominal interest rate. To account for this new feature of the model, we exploit the algorithm proposed by Guerrieri and Iacoviello (2015) to deal with occasionally binding constraints. Their tool is built on the insight that occasionally binding constraints, like the zero lower bound, can be handled as different regimes of the same model: under one regime the occasionally binding constraint is slack and a rational

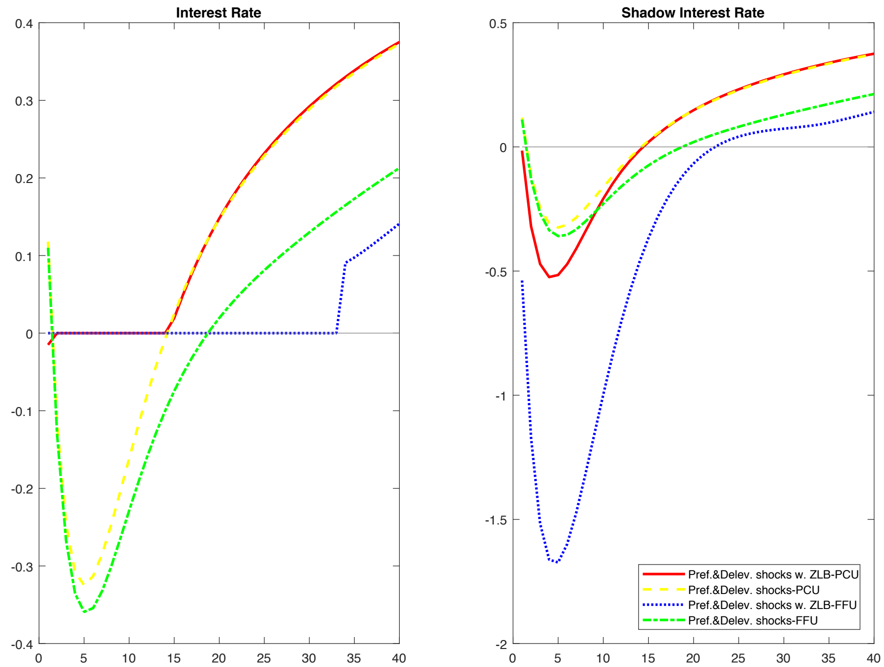
Figure 8: Deleveraging with and without the ZLB - Pure Currency Union vs Full Fiscal Union



expectations solution exists, while under the other regime the same constraint is binding and the model might yield multiple solutions. However, if a shock moves the model from the slack regime to the binding one, it must return to the original regime in a finite time horizon. The model is linearized around the non-stochastic steady state of the slack regime and the piecewise linear solution method proposed involves a first order approximation of the model around the same point for each regime. Since the dynamics in a regime may crucially depend on its expected duration, which in turn depends on the state vector, the solution is highly non-linear.

This method is not able to capture the precautionary behavior linked to the possibility that a constraint may become binding in the future as a result of shocks that have not yet realized. Henceforth, differently form the analysis in previous sections, we assume that the economy is already facing a recession due to a large negative preference shock in country F, so that the nominal interest rate hits the zero lower bound from the first period onwards for around 15-35 quarters, depending on the scenario. It is worth noting that the deleveraging shock alone, despite its size, does not push

Figure 9: Deleveraging with and without the ZLB - Pure Currency Union vs Full Fiscal Union



To show the actual constraint imposed by the zero lower bound, both the nominal interest rate and the shadow interest rate (thicker lines) are reported in levels rather than in deviation from steady state.

any of the economies into a liquidity trap.

Figure 8 compares the impulse responses after a shock to the government debt target in country F with and without the zero lower bound constraint, in both the Pure Currency Union scenario and the Full Fiscal Union one. Figure 9 shows the nominal interest rate and the shadow interest rate from which one can see the duration of the liquidity trap.

In the absence of the Zero Lower Bound constraint, analyzing the effect of the shock to the government debt target in conjunction with a preference shock (dashed yellow line and dash-dotted green line) or without it, the dynamics are quite similar because the deleveraging shock is the main driver of the economy, although they are amplified in the former case. This is why the results obtained here are comparable with those obtained in the previous sections. However, the deflationary pressures observed also in country H after a preference shock bring the nominal interest rate into negative territory. Comparing these results with the case with a Zero Lower Bound constraint (solid red line and dotted blue line), we can observe that the deflationary pressures are higher in both countries in the presence of the constraint in both scenarios. Since the impact on country F's PPI inflation is larger than that on country H's PPI inflation, the negative effects of the additional deflationary pressures are amplified by the higher volatility in the terms of trade and thus in net exports. Furthermore, the increase in the real interest rate (given by the binding Zero Lower Bound) strongly smooths the increase in consumption in country H.

In both scenarios with the ZLB, despite the tight deleveraging rule, government debt in country F grows in real terms for some quarters, because the deflationary pressures are not offset by the Central Bank, which is constrained. As argued in Erceg and Lindé (2014), when the Zero Lower Bound is binding the government deficit decreases by less in response to spending cuts, if the tax base responds more to swings in output. Looking at Figure 8, it is possible to observe that the temporary increase in government debt is more pronounced in the Full Fiscal Union scenario because government consumption in both countries becomes highly pro-cyclical, creating further deflation compared to the Pure Currency Union scenario. Therefore, as shown in Figure 9 the liquidity trap lasts longer in the Full Fiscal Union scenario (about 35 quarters) than in the Pure Currency Union one (about 15 quarters). Furthermore, since in the former scenario the deflationary pressures are highly exacerbated, the interest rate is stuck at zero in the presence of the constraint for more than it is in negative territory in the absence of the constraint, making the overall process self-reinforcing. This is true in the Full Fiscal Union scenario, while in the Pure Currency Union scenario the timing of the exit from zero or negative rates is the same. Moreover, the fact that in the Full Fiscal Union scenario the interest rate becomes positive later in the presence of the Zero Lower Bound constraint than without it, increases the persistence of most variables away from the steady state, as the Central Bank is not able to stabilize enough the economy because it cannot respond to the deflationary pressures for a longer time than expected. The difference in the movements in prices between the PCU and the FFU scenarios is much greater in the presence of the ZLB constraint than without it, while the difference in the movements in GDP in both countries is smaller.

5 Welfare Analysis based on an ad hoc Loss Function

As a further step of the analysis, we compare the stabilization properties of the fiscal policy scenarios and of the deleveraging instruments by means of an ad hoc loss function. Since the main focus of this work is to evaluate the role of fiscal policy as a stabilizer of the economy, we prefer to rely on a quadratic loss function rather than to look at Consumption Equivalent Variations (CEV). Specifically, fiscal policy has a stabilizing function for the real economy similar to the role played by monetary policy for prices and, hence, together they aim to reduce both the inflation gap and the output gap. As argued in Blanchard, Erceg and Lindé (2015), since our model and calibration assume a large resource slack (negative net exports) between the Periphery (country F) and Germany (country H), the gains in terms of consumption and unemployment related to closing the output gap are underestimated by utility-based measures.

Using a standard quadratic loss function, the policymakers are assumed to care only about minimizing the square of the output gap and of the inflation gap in both regions. Each region's loss function is, hence, simply the sum of the square of the inflation gap and the square of the output gap, with weights 3 and 1 respectively. The overall loss function is given by the weighted average of each region's loss function:

$$Loss = \sum_{j=0}^{\infty} \beta^j \left\{ h \left[(\hat{\pi}_{t+j})^2 + \frac{1}{3}(\hat{Y}_{t+j})^2 \right] + (1-h) \left[(\hat{\pi}^*_{t+j})^2 + \frac{1}{3}(\hat{Y}^*_{t+j})^2 \right] \right\} \tag{5.0.1}$$

The welfare costs for each scenario are computed as the difference in the conditional mean of the second order approximation of the loss function around the non-stochastic steady state with respect to the scenario with the lowest loss.

Table 2: Welfare Costs: Comparison of Fiscal Scenarios by Instrument

Welfare Costs based on ad hoc loss function			
Fiscal Instrument: Government Consumption			
	Country H	Country F	Average
PCU	216.3%	160.7%	188.1%
CCU	9.33%	9.38%	9.36%
FFU *	0%	0%	0%
Fiscal Instrument: Government Transfers			
	Country H	Country F	Average
PCU	93.55%	196.9%	140.7%
CCU	22.99%	49.51%	35.09%
FFU*	0%	0%	0%
Fiscal Instrument: Tax on Sales and on Wages			
	Country H	Country F	Average
PCU	25.02%	82.64%	45.19%
CCU *	0%	0%	0%
FFU	50.20%	62.72%	54.58%

Welfare Costs are computed as $\frac{Loss_a - Loss_b}{Loss_b}$, with b the scenario featuring the lowest loss for the selected fiscal instrument (indicated with *)

Table 2 compares the welfare costs in terms of the loss function for the three fiscal policy scenarios in the case in which country F has a level of government debt-to-GDP above the target of 60% and follows a linear deleveraging scheme. This analysis aims at evaluating the risk-sharing properties of different degrees of fiscal policy coordination after a shock to the government debt target, which is followed by a prolonged period of deleveraging and austerity, also with demand or supply shocks or with the Zero Lower Bound. The results reported for the three alternative fiscal instruments for deleveraging support the analysis done in section 4.3. Indeed, if the economy is hit by a sizable shock to the government debt target, targeting the net exports gap is welfare improving with respect to targeting the output gap, for all instruments used for deleveraging. Additionally, consolidating budget constraints decreases welfare only when the deleveraging costs are covered by taxes, overcoming the benefits of targeting the net exports gap. This is mainly due to the fact that in the Full Fiscal Union scenario fiscal policy instruments move jointly, which implies a reduction in the tax rates in both countries when deleveraging with government transfers or government consumption. On the other hand, tax rates persist above the steady state for a long period, like in the Pure Currency Union scenario, if the overall cost of debt stabilization and debt reduction is supported by taxes.

Table 3: Welfare Costs: Comparison of Fiscal Instruments by Scenario

Welfare Costs based on ad hoc loss function			
Fiscal Scenario: Pure Currency Union			
	Country H	Country F	Average
Gov. Cons.	292.3%	323.3%	305.9%
Gov. Tr.	211.0%	409.1%	298.2%
Taxes*	0%	0%	0%
Fiscal Scenario: Coordinated Currency Union			
	Country H	Country F	Average
Gov. Cons.	69.53%	224.3%	123.7%
Gov. Tr.	147.1%	368.2%	224.5%
Taxes*	0%	0%	0%
Fiscal Scenario: Full Fiscal Union			
	Country H	Country F	Average
Gov. Cons.	3.23%	82.20%	32.33%
Gov. Tr.	33.75%	92.46%	55.38%
Taxes*	0%	0%	0%

Welfare Costs are computed as $\frac{Loss_a - Loss_b}{Loss_b}$, with b the instrument featuring the lowest loss for the selected fiscal scenario (indicated with *)

As argued above, it is also possible to notice that the welfare costs from consolidating budget constraints vanish if deleveraging is achieved by using government consumption or transfers. To deeply investigate this phenomenon, Table 3 reports the welfare costs of using a specific fiscal instrument for deleveraging in each of the three scenarios for fiscal policy coordination. It is possible to notice that if the cost of deleveraging, along with the cost of government consumption, is financed by distortionary taxes, in all three scenarios, we achieve the most stabilization. As discussed in Section 4.2, if deleveraging is achieved with government consumption or transfers, it adds a source of deflationary pressure to the economy, inducing large movements in the interest rate and in the terms of trade. Therefore, if deleveraging is performed by moving government consumption or transfers, they fluctuate largely compared to using taxes to deleverage and this destabilizes further output in both countries, if the government budget is not consolidated. In particular, given the direct impact of government consumption on output, its dynamics are even more amplified when government consumption is employed to deleverage and government transfers target either the output gap or the net exports gap. It is also interesting to notice that the welfare costs of deleveraging with instruments other than taxes are much lower in the Full Fiscal Union scenario with respect to the Pure Currency Union scenario. In this case the effects of the tax rates on inflation are reduced and, despite the interest rate fluctuates less, GDP in both countries fluctuates more due to the higher volatility of the terms of trade.

In conclusion, if the economy with a large resource slack faces a prolonged period of deleveraging, a high degree of fiscal policy coordination is beneficial for the economy. However, the consolidation of budget constraints might be only a slight improvement, or be even costly, in terms of stabilization. This is especially true if taxes are used both to keep the budget balanced in country H and to reduce the government debt in country F, as the gains from using taxes rather than other fiscal instruments strongly decreases in the Full Fiscal Union scenario. Finally, although taxes create more distortions than government consumption or transfers when deleveraging, their movements reduce the overall volatility in the economy in all fiscal policy scenarios.

6 Conclusions and Possible Extensions

This research was conducted to assess the effects of deleveraging in the Eurozone, as requested by the European Commission, and the stabilization properties of different fiscal instruments for deleveraging and different scenarios for fiscal policy coordination, to bring to the proper government debt management in a Currency Union. We build a Two-Country (Germany vs. the Periphery) New-Keynesian DSGE model of a Currency Union with incomplete international financial markets, to study the effects of government debt deleveraging in the Periphery and its spillovers towards Germany.

The effects of international spillovers are amplified when the two countries do not coordinate their fiscal policies (Pure Currency Union scenario), because taxes and thus prices move in opposite directions across countries, reducing deflation, but moving more relative prices and bringing net exports lower, pushing GDP to persist longer away from its steady state in both countries. As a consequence, we find that coordinating on the net exports gap and (to a minor extent) consolidating budget constraints across countries when deleveraging generally provides more stabilization.

We complement our analysis by computing welfare costs based on an ad hoc loss function, by fiscal scenario and by deleveraging instrument. If a country with a large resource slack (the Periphery) faces a prolonged period of deleveraging, coordinating fiscal policy yields welfare gains. However, the additional consolidation of budget constraints might induce welfare costs.

As a robustness check, we also consider the implications of the Zero Lower Bound in the deleveraging dynamics and consequent spillovers. We observe that deflationary pressures are stronger in both countries (but more in the Periphery), because the Central Bank cannot counteract deflation in the presence of the constraint, and this brings to higher volatility in the terms of trade and in net exports. Furthermore, the liquidity trap lasts longer in the Full Fiscal Union scenario because government consumption becomes highly procyclical especially in country H, creating further deflation.

We also document the benefits of responding to the net exports gap when Germany is hit by a negative technology shock. By comparing the dynamics after such a shock, with and without deleveraging, we disentangle the effect of the limitations imposed on fiscal policy stabilization by the fiscal consolidation from the effect of the deleveraging shock itself. In every fiscal policy scenario, the overall volatility and persistence of the economy is higher when deleveraging, compared to balancing the budget. This is because when a productivity shock hits the economy and one country is in a relevant deleveraging process, the dynamics in the economy are driven more by the deflationary pressure induced by the deleveraging shock, than by the productivity shock itself.

The best instrument for deleveraging are distortionary taxes in all fiscal policy scenarios, because they counteract the deflationary effect of the deleveraging shock more than other instruments. This result partially contrasts the findings in the previous literature, which point out to the fact that tax reductions and expenditure cuts are the best fiscal policy mix, but, as stated in Coenen, Mohr and Straub (2008), tax based consolidations make the reactions of GDP, inflation and the terms of trade smoother.

By studying different deleveraging schemes, we find that the backloading scheme is much more stabilizing than both the linear and frontloading ones in all scenarios, while the linear deleveraging scheme produces only slightly more stabilization than the frontloading one. This shows that the timing of deleveraging matters, as reducing government debt more gradually over time reduces overall volatility in the economy.

Although our results are robust to different calibrations of the key parameters (see Appendix B), our model has nonetheless some shortcomings, which entail possible future avenues of research. First, our model focuses on a very specific design of fiscal policy, and one can consider coordination strategies which are different from ours. Second, a more complex assumption on the structure of international financial markets might change the amount of private risk-sharing across countries and the international transmission of shocks. Finally, we do not look at the distributional consequences of fiscal consolidations, which might enhance the stabilization properties of expenditure based fiscal consolidations, if the transfers are finalized at reducing inequalities.

Our policy prescriptions for the Eurozone are to reduce government debt more gradually over time and less during recessions, to do so using distortionary taxes, while concentrating on reducing international demand imbalances and maybe creating some form of fiscal union.

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A Mathematical Appendix

As a tool for students and researchers studying models of this kind for the first time, I provide as many mathematical derivations of the model as possible here in the Appendix. This is meant for learning purposes, especially concerning complex models with cumbersome mathematical derivations.

A.1 Equilibrium Conditions

Here we collect all the equilibrium conditions of the full model, differentiating between a pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union and between different policy rules.
The equilibrium conditions of the model are grouped into the following blocks:

Aggregate Demand Block
The aggregate demand block is composed of aggregate demand in both countries H:

$$Y_t = \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha)C_t + \alpha^*(\mathcal{S}_t)^{\eta}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{\eta}{1-\eta}} C_t^*\right] + G_t \tag{A.1.1}$$

and F:

$$Y_t^* = \left[1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}\right]^{\frac{\eta}{1-\eta}} \left[(1-\alpha^*)C_t^* + \alpha(\mathcal{S}_t)^{-\eta}\left(\frac{1-\alpha^*+\alpha^*(\mathcal{S}_t)^{\eta-1}}{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}\right)^{\frac{\eta}{\eta-1}} C_t\right] + G_t^* \tag{A.1.2}$$

while the evolution of private consumption is given by the households' Euler Equation in countries H:

$$\frac{1}{1+i_t} = \beta E_t\left\{\frac{\xi_{t+1}}{\xi_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{\Pi_{t+1}}\right\} \tag{A.1.3}$$

and F:

$$\frac{1-\delta_t}{1+i_t} = \beta E_t\left\{\frac{\xi^*_{t+1}}{\xi^*_t}\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\frac{1}{\Pi^*_{t+1}}\right\} \tag{A.1.4}$$

while the relationship between CPI inflation and PPI inflation is given by:

$$\Pi_t = \Pi_{H,t}\left[\frac{1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}}{1-\alpha+\alpha(\mathcal{S}_{t-1})^{1-\eta}}\right]^{\frac{1}{1-\eta}} \tag{A.1.5}$$

in country H and:

$$\Pi_t^* = \Pi_{H,t}^*\left[\frac{1-\alpha^*+\alpha^*\mathcal{S}_t^{\eta-1}}{1-\alpha^*+\alpha^*(\mathcal{S}_{t-1})^{\eta-1}}\right]^{\frac{1}{1-\eta}} \tag{A.1.6}$$

in country F, and the evolution of the terms of trade is given by:

$$\mathcal{S}_t = \frac{\Pi^*_{H,t}}{\Pi_{H,t}} \mathcal{S}_{t-1} \tag{A.1.7}$$

while the exogenous demand shocks evolve according to:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \tag{A.1.8}$$

$$\xi^*_t = (\xi^*_{t-1})^{\rho^*_\xi} e^{\varepsilon_t} \tag{A.1.9}$$

Aggregate Supply Block

The aggregate supply block is composed of the aggregate supply equation for country H:

$$\left(\frac{1 - \theta (\Pi_{H,t})^{\varepsilon - 1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K_t}{F_t} \tag{A.1.10}$$

where:

$$K_t = \xi_t (C_t)^{-\sigma} Y_t MC_t + \beta \theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon+1}}{\Pi_{t+1}} K_{t+1} \right\} \tag{A.1.11}$$

$$F_t = \xi_t (C_t)^{-\sigma} Y_t (1 - \tau^s_t) + \beta \theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon}}{\Pi_{t+1}} F_{t+1} \right\} \tag{A.1.12}$$

and marginal cost in country H is given by:

$$MC_t = \frac{(Y_t)^\varphi (d_t)^\varphi (C_t)^\sigma}{(1 - \tau^w_t)(A_t)^{1+\varphi} (h)^{\varphi+\sigma}} \left[1 - \alpha + \alpha (\mathcal{S}_t)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{A.1.13}$$

and the aggregate supply equation for country F:

$$\left(\frac{1 - \theta^* (\Pi^*_{H,t})^{\varepsilon - 1}}{1 - \theta^*} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K^*_t}{F^*_t} \tag{A.1.14}$$

where:

$$K^*_t = \xi^*_t (C^*_t)^{-\sigma} Y^*_t MC^*_t + \beta \theta^* E_t \left\{ \frac{(\Pi^*_{H,t+1})^{\varepsilon+1}}{\Pi^*_{t+1}} K^*_{t+1} \right\} \tag{A.1.15}$$

$$F^*_t = \xi^*_t (C^*_t)^{-\sigma} Y^*_t (1 - \tau^{*s}_t) + \beta \theta^* E_t \left\{ \frac{(\Pi^*_{H,t+1})^{\varepsilon}}{\Pi^*_{t+1}} F^*_{t+1} \right\} \tag{A.1.16}$$

and marginal cost in country F is given by:

$$MC^*_t = \frac{(Y^*_t)^\varphi (d^*_t)^\varphi (C^*_t)^\sigma}{(1 - \tau^{*w}_t)(A^*_t)^{1+\varphi} (1-h)^{\varphi+\sigma}} \left[1 - \alpha^* + \alpha^* (\mathcal{S}_t)^{\eta-1} \right]^{\frac{1}{1-\eta}} \tag{A.1.17}$$

while the evolution of price dispersion is given by:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^{\varepsilon} + (1-\theta)\left[\frac{1-\theta(\Pi_{H,t})^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.1.18}$$

for country H, and:

$$d_t^* = \theta^* d_{t-1}^*(\Pi_{H,t}^*)^{\varepsilon} + (1-\theta^*)\left[\frac{1-\theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1-\theta^*}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.1.19}$$

for country F, while the levels of technology evolve exogenously according to:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \tag{A.1.20}$$

$$A_t^* = (A_{t-1}^*)^{\rho_a^*} e^{\varepsilon_t} \tag{A.1.21}$$

Net Exports, Net Foreign Assets and the Balance of Payments

Real Net Exports for country H are given by:

$$\widetilde{NX}_t = Y_t - \left[1-\alpha+\alpha(\mathcal{S}_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} C_t - G_t \tag{A.1.22}$$

Real Net Foreign Assets for country H are given by:

$$\widetilde{NFA}_t = \tilde{B}_{F,t} \tag{A.1.23}$$

The real Balance of Payments for country H is given by:

$$\widetilde{BP}_t = \widetilde{NX}_t + \frac{i_{t-1}+\delta_{t-1}}{1-\delta_{t-1}}\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} \tag{A.1.24}$$

so that real Net Foreign Assets for country H evolve according to:

$$\widetilde{NFA}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t = \frac{1+i_{t-1}}{1-\delta_{t-1}}\frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{NX}_t \tag{A.1.25}$$

while the transaction cost is given by:

$$\delta_t \equiv (1-\rho_\delta)\delta^B\left(\frac{B_{t-1}^{*G}}{P_{H,t-1}^* Y_{t-1}^*} - \frac{B^{*G}}{P_H^* Y^*}\right) + \rho_\delta \delta_{t-1} \tag{A.1.26}$$

Monetary Policy

Monetary policy sets the nominal interest rate following the rule:

$$\beta(1+i_t) = \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} [\beta(1+i_{t-1})]^{\rho_i} \tag{A.1.27}$$

where union-wide CPI inflation is defined by:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \tag{A.1.28}$$

while in the alternative Zero Lower Bound scenario it follows the rule:

$$\begin{aligned} i_t &= \max\{\tilde{i}_t, 0\} \\ \beta(1+\tilde{i}_t) &= \left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\pi(1-\rho_i)} \left[\beta(1+\tilde{i}_{t-1})\right]^{\rho_i} \end{aligned} \tag{A.1.29}$$

Fiscal Policy in a Pure Currency Union - Transfer Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.30}$$

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.31}$$

while using real transfers to deleverage government debt in country F and balance the budget in country H, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{A.1.32}$$

and varying the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) Y_t^* - (\tau^{*s} + \tau^{*w} MC^*) Y^* = G_t^* - G^* \tag{A.1.33}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{A.1.34}$$

$$G_t^* + \tilde{T}_t^* + \frac{i_{t-1} + \delta_{t-1}}{1-\delta_{t-1}} \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.35}$$

Fiscal Policy in a Pure Currency Union - Consumption Scenario

Fiscal policy sets real transfers following the rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{A.1.36}$$

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{A.1.37}$$

while using government consumption to deleverage government debt in country F and balance the budget in country H, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{A.1.38}$$

and varying the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) Y_t^* - (\tau^{*s} + \tau^{*w} MC^*) Y^* = \tilde{T}_t^* - \tilde{T}^* \tag{A.1.39}$$

Fiscal Policy in a Pure Currency Union - Distortionary Tax Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y}\right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.40}$$

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*}\right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.41}$$

while using equally taxes on labour income and on firm sales to deleverage government debt in country F and balance the budget in country H, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{A.1.42}$$

and keeping real transfers constant, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tilde{T}_t = \tilde{T} \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad \tilde{T}_t^* = \tilde{T}^* \tag{A.1.43}$$

Fiscal Policy in a Coordinated Currency Union - Transfer Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.44}$$

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.45}$$

while using real transfers to deleverage government debt in country F and balance the budget in country H, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{A.1.46}$$

and varying the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) Y_t^* - (\tau^{*s} + \tau^{*w} MC^*) Y^* = G_t^* - G^* \tag{A.1.47}$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1} \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \tag{A.1.48}$$

$$G_t^* + \tilde{T}_t^* + \frac{i_{t-1} + \delta_{t-1}}{1 - \delta_{t-1}} \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.49}$$

Fiscal Policy in a Coordinated Currency Union - Consumption Scenario

Fiscal policy sets real transfers following the rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{A.1.50}$$

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{A.1.51}$$

while using government consumption to deleverage government debt in country F and balance the budget in country H, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{A.1.52}$$

and varying the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) Y_t^* - (\tau^{*s} + \tau^{*w} MC^*) Y^* = \tilde{T}_t^* - \tilde{T}^* \tag{A.1.53}$$

Fiscal Policy in a Coordinated Currency Union - Distortionary Tax Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.54}$$

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.55}$$

while using equally taxes on labour income and on firm sales to deleverage government debt in country F and balance the budget in country H, through the debt rules:

$$\tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \tag{A.1.56}$$

and keeping real transfers constant, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tilde{T}_t = \tilde{T} \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \qquad \tilde{T}_t^* = \tilde{T}^* \tag{A.1.57}$$

Fiscal Policy in a Full Fiscal Union - Transfer Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.58}$$

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.59}$$

while using real transfers equally in both countries to deleverage the government debt of country F, while country H maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \tilde{T}_t - \tilde{T} = \tilde{T}_t^* - \tilde{T}^* \tag{A.1.60}$$

and varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^w - \tau^w \qquad \tau_t^{*s} - \tau^{*s} = \tau_t^s - \tau^s \tag{A.1.61}$$

$$(\tau_t^s + \tau_t^w MC_t d_t)Y_t + (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*)S_t Y_t^* - (\tau^s + \tau^w MC)Y - (\tau^{*s} + \tau^{*w} MC^*)Y^* = G_t + G_t^* - G - G^* \tag{A.1.62}$$

with the following consolidated budget constraint:

$$G_t + \tilde{T}_t + S_t(G_t^* + \tilde{T}_t^*) + i_{t-1}\frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + \frac{i_{t-1} + \delta_{t-1}}{1 - \delta_{t-1}}\frac{S_{t-1}\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} =$$
$$(\tau_t^s + \tau_t^w MC_t d_t)Y_t + (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*)S_t Y_t^* + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + S_t \tilde{B}_t^{*G} - \frac{S_{t-1}\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \tag{A.1.63}$$

Fiscal Policy in a Full Fiscal Union - Consumption Scenario

Fiscal policy sets real transfers following the rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}}\right)^{\rho_t} e^{\varepsilon_t} \tag{A.1.64}$$

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*}\right)^{\rho_t^*} e^{\varepsilon_t} \tag{A.1.65}$$

while using government consumption equally in both countries to deleverage the government debt of country F, while country H maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^*\left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad G_t - G = G_t^* - G^* \tag{A.1.66}$$

and varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^w - \tau^w \qquad \tau_t^{*s} - \tau^{*s} = \tau_t^s - \tau^s \tag{A.1.67}$$

$$(\tau_t^s + \tau_t^w MC_t d_t)Y_t + (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*)S_t Y_t^* - (\tau^s + \tau^w MC)Y - (\tau^{*s} + \tau^{*w} MC^*)Y^* = T_t + T_t^* - T - T^* \tag{A.1.68}$$

Fiscal Policy in a Full Fiscal Union - Distortionary Tax Scenario

Fiscal policy sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}}\right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G}\right)^{\rho_g} e^{\varepsilon_t} \tag{A.1.69}$$

$$\frac{G_t^*}{G^*} = \left(\frac{S\,\widetilde{NX}_t}{S_t\,\widetilde{NX}}\right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*}\right)^{\rho_g^*} e^{\varepsilon_t} \tag{A.1.70}$$

while keeping real transfers constant and varying equally across countries the tax rates on labour income and firm sales to deleverage the government debt of country F, while country H maintains its government debt constant, through the debt rules:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G}\right) \qquad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \qquad \tilde{T}_t = \tilde{T} \qquad \tilde{T}_t^* = \tilde{T}^* \tag{A.1.71}$$

and also varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure, through the tax rules:

$$\tau_t^w - \tau^w = \tau_t^s - \tau^s \qquad \tau_t^{*w} - \tau^{*w} = \tau_t^w - \tau^w \qquad \tau_t^{*s} - \tau^{*s} = \tau_t^s - \tau^s \tag{A.1.72}$$

We can now define an equilibrium for the Currency Union.

Definition 1 (Equilibrium)**.** *An Imperfectly competitive equilibrium is a sequence of stochastic processes:*
$\mathcal{X}_t \equiv \{Y_t, Y_t^*, C_t, C_t^*, \Pi_{H,t}, \Pi_{H,t}^*, \Pi_t, \Pi_t^*, \Pi_t^U, S_t, K_t, K_t^*, F_t, F_t^*, MC_t, MC_t^*, d_t, d_t^*, \widetilde{NX}_t, \widetilde{NFA}_t, \widetilde{CA}_t\}$
and exogenous disturbances:
$\mathcal{Z}_t \equiv \{\xi_t, \xi_t^*, A_t, A_t^*\}$ *satisfying equations A.1.1 through A.1.25 and the definition of union-wide inflation A.1.28, given initial conditions:*
$\mathcal{I}_{-1} \equiv \{C_{-1}, C_{-1}^*, \Pi_{H,-1}, \Pi_{H,-1}^*, S_{-1}, d_{-1}, d_{-1}^*, \widetilde{NFA}_{-1}\}$*, plus monetary and fiscal policies:*
$\mathcal{P}_t \equiv \{i_t, G_t, G_t^*, \tilde{T}_t, \tilde{T}_t^*, \tau_t^s, \tau_t^{*s}, \tau_t^w, \tau_t^{*w}, \tilde{B}_t^G, \tilde{B}_t^{*G}\}$ *specified in equation A.1.27 or A.1.29 for monetary policy and in equations A.1.30 through A.1.72 for the various specifications of fiscal policy, for* $t \geq 0$*.*

A.2 The Steady State

We describe the symmetric (in terms of per capita consumption and prices) non-stochastic steady state with constant government debt and zero inflation. We focus on the perfect foresight steady state and equilibrium deviations from it, given by different shocks. *Perfect Foresight* is a viable assumption because, despite the uncertainty to which price-setters are subject, it disappears in the aggregate due to the further assumption that there is a large number (more accurately, a continuum) of firms, as explained in Calvo (1983).
The symmetric non-stochastic steady state with constant government debt and zero inflation is defined by the following assumptions and equations.
All shocks are constant at their long-run levels of 1:

$$\xi = \xi^* = A = A^* = 1 \tag{A.2.1}$$

There is no inflation and no price dispersion:

$$\Pi_H = \Pi_H^* = \Pi = \Pi^* = \Pi^U = 1 \implies d = d^* = 1 \tag{A.2.2}$$

The terms of trade and the real exchange rate are equal to 1:

$$S = 1 \implies Q = 1 \tag{A.2.3}$$

Per-capita consumption is equal across countries:

$$\frac{C}{h} = \frac{C^*}{1-h} \tag{A.2.4}$$

Aggregate demand in each country is given by:

$$Y = (1-\alpha)\,C + \alpha^* C^* + G \tag{A.2.5}$$

$$Y^* = (1-\alpha^*)\,C^* + \alpha C + G^* \tag{A.2.6}$$

Combining the previous equations we can derive per-capita consumption in each country as a function of output and government spending and equate the two to derive an equation linking output and government spending in the two countries:

$$Y = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h}\left[Y^* - G^*\right] + G \tag{A.2.7}$$

From the Euler Equations:

$$\frac{1}{1+i} = \beta \implies i = \frac{1}{\beta} - 1 \tag{A.2.8}$$

Recalling marginal costs in steady state from price-setting:

$$MC = \frac{\varepsilon - 1}{\varepsilon}(1-\tau^s) \tag{A.2.9}$$

$$MC^* = \frac{\varepsilon - 1}{\varepsilon}(1-\tau^{*s}) \tag{A.2.10}$$

Marginal costs are also given by labour market equilibrium:

$$MC = \frac{(Y)^\varphi (C)^\sigma}{(1-\tau^w)(h)^{\varphi+\sigma}} \tag{A.2.11}$$

$$MC^* = \frac{(Y^*)^\varphi (C^*)^\sigma}{(1-\tau^{*w})(1-h)^{\varphi+\sigma}} \tag{A.2.12}$$

Equating the two marginal cost expressions for each country yields consumption in terms of output:

$$C = \left[\frac{\varepsilon - 1}{\varepsilon}\frac{(1-\tau^s)(1-\tau^w)(h)^{\varphi+\sigma}}{(Y)^\varphi}\right]^{\frac{1}{\sigma}} \tag{A.2.13}$$

$$C^* = \left[\frac{\varepsilon - 1}{\varepsilon} \frac{(1-\tau^{*s})(1-\tau^{*w})(1-h)^{\varphi+\sigma}}{(Y^*)^{\varphi}}\right]^{\frac{1}{\sigma}} \tag{A.2.14}$$

Deriving per-capita consumption in the two countries and equating the two yields an equation linking output in the two countries:

$$Y^* = \frac{1-h}{h}\left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^{s})(1-\tau^{w})}\right]^{\frac{1}{\varphi}} Y \tag{A.2.15}$$

In steady state real net exports are given by:

$$\widetilde{NX} = Y - C - G \tag{A.2.16}$$

while real net foreign assets are:

$$\widetilde{NFA} = \tilde{B}_F \tag{A.2.17}$$

The real balance of payments is given by:

$$\widetilde{BP} = \widetilde{NX} + \left(\frac{1}{\beta} - 1\right)\widetilde{NFA} \tag{A.2.18}$$

while from the budget constraints of households and governments, or equivalently from the evolution of net foreign assets:

$$\widetilde{NFA} = \widetilde{NFA} + \widetilde{BP} \tag{A.2.19}$$

which implies that in steady state the balance of payments must be zero and so net exports pin down net foreign assets:

$$\widetilde{BP} = 0 \implies \widetilde{NX} = -\left(\frac{1}{\beta} - 1\right)\widetilde{NFA} \tag{A.2.20}$$

while the transaction cost in steady state is zero, because debt is constant at steady state level and there are no shocks:

$$\delta = 0 \tag{A.2.21}$$

The household budget constraints in steady state for countries H and F are given by:

$$C = \left(\frac{1}{\beta} - 1\right)(\tilde{B}_H + \tilde{B}_F) + \tilde{T} + Y(1 - \tau^s - \tau^w MC) \tag{A.2.22}$$

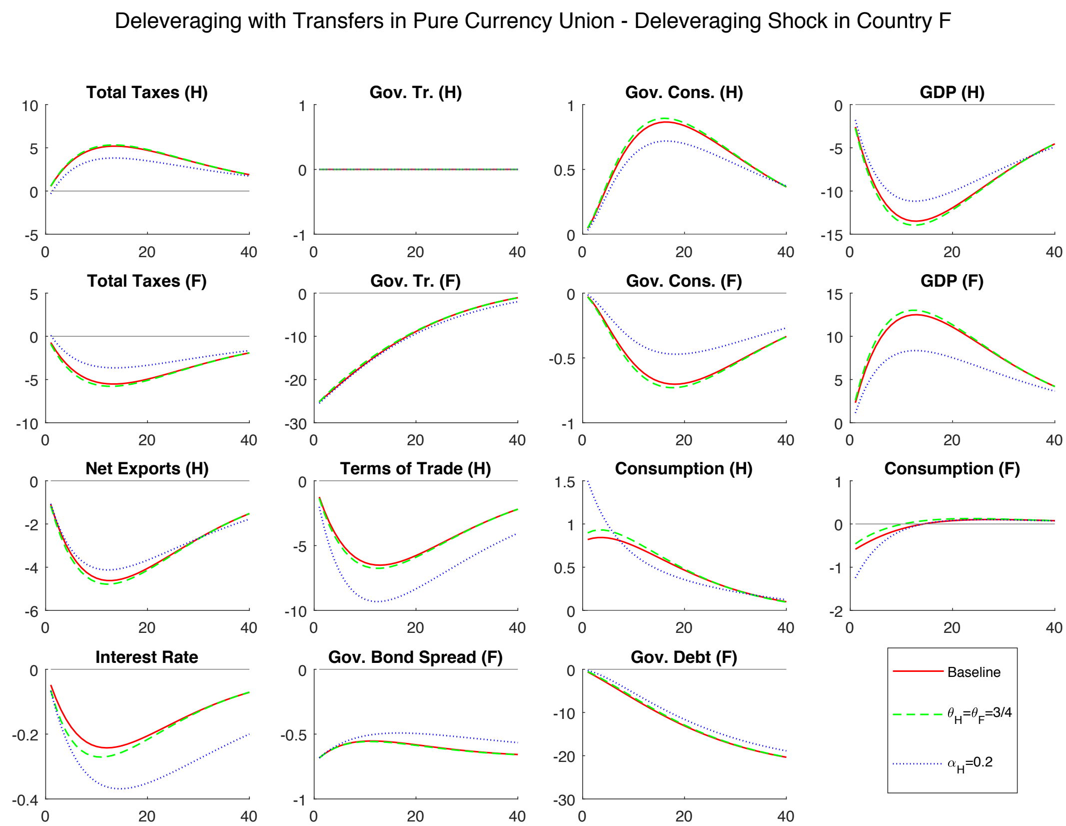
$$C^* = \left(\frac{1}{\beta} - 1\right)\tilde{B}^*_F + \tilde{T}^* + Y^*(1 - \tau^{*s} - \tau^{*w} MC^*) \tag{A.2.23}$$

Instead the government budget constraints of the two countries in steady state read:

$$G + \tilde{T} + \left(\frac{1}{\beta} - 1\right)\tilde{B}^G = Y(\tau^s + \tau^w MC) \tag{A.2.24}$$

$$G^* + \tilde{T}^* + \left(\frac{1}{\beta} - 1\right)\tilde{B}^{*G} = Y^*(\tau^{*s} + \tau^{*w} MC^*) \tag{A.2.25}$$

Figure 10: Sensitivity of Deleveraging to θ and α - Pure Currency Union - Government Transfers



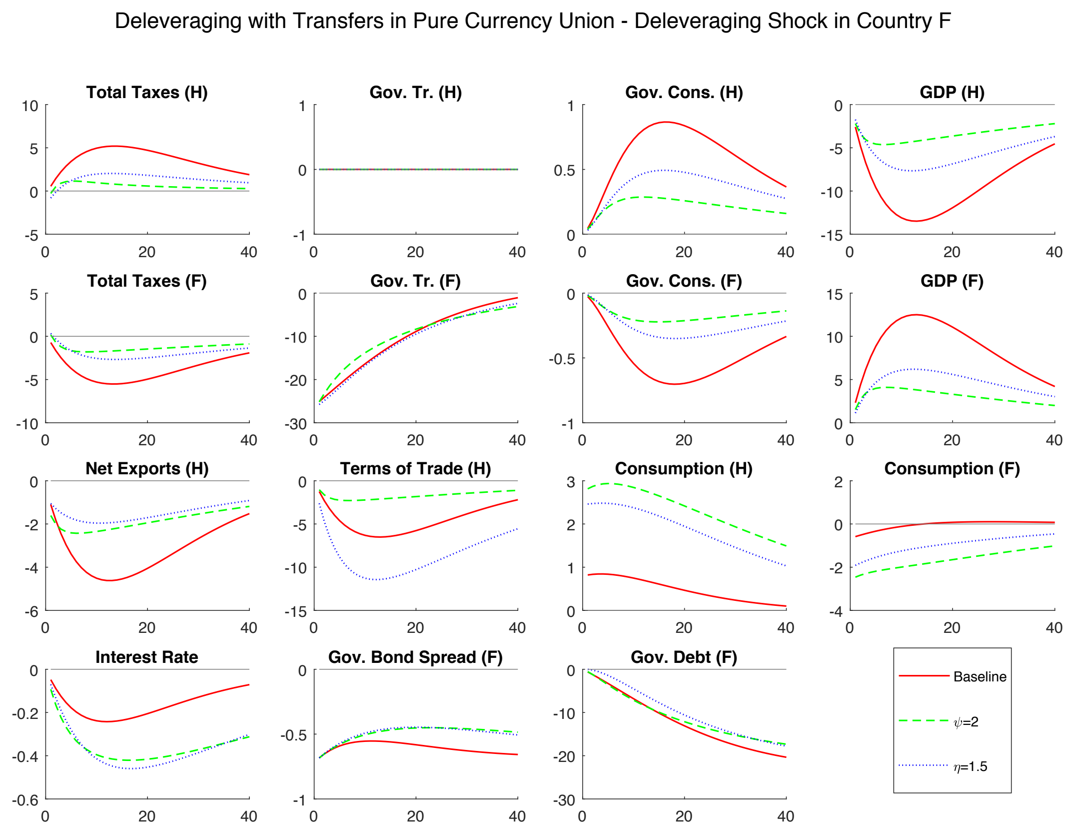
B Sensitivity Analysis

Looking at the previous literature, such as Coenen et al. (2012), it turns out that the strength of international spillovers and the effect of having a supranational policy rate is affected by a few key parameters. For this reason, we decide to perform a robustness check to prove that our results hold even if some features of the model are different.

Since international spillovers mainly depend on the difference in price stickiness and on the difference in the openness to trade of the two countries, Figure 10 reports the impulse responses after a shock to the government debt target in the Pure Currency Union scenario in which country F's deleveraging is financed with government transfers. The Figure compares the impulse responses in the baseline calibration (solid red line) to the case in which price stickiness is the same in both countries, ($\theta_H = \theta_F = 3/4$, as in the calibration for country H, dashed green line) and to the case in which both economies are almost closed (dotted blue line).

We observe that openness drives our results more than the difference in price rigidity because

Figure 11: Sensitivity of deleveraging to φ and η - Pure Currency Union - Government Transfers



the differences between the dashed green lines and the solid red lines are negligible, while several variables fluctuate less if the economies are closed ($\alpha_H = 0.2$ and $\alpha_F = 0.15$). However, spillovers are sizeable even with a high home bias in consumption (or low openness to trade). In this case, consumption, the terms of trade and the interest rate are more volatile, while GDP in both countries is more stable. The higher volatility in consumption and in the terms of trade does not translate into a higher volatility in GDP because net exports are much less sensitive to movements in price differentials when the economies are almost closed.

However, the literature (see Hjortsø (2016) and Galí and Gertler (2010)) shows that the elasticity of substitution between domestic and imported goods is a key parameter for any open economy DSGE model, since it affects how demand for different goods responds to relative prices. This parameter, together with the Frisch elasticity of labour supply and the elasticity of intertemporal substitution, determines whether a higher trade openness has large or small effects on output and inflation, because all these parameters influence the slope of the labour supply curve in the same direction as openness does. Indeed, the more substitutable the goods (or the larger the share of imported goods) the flatter the curve and, hence, consumers change more their domestic and imported quantities in order to smooth aggregate consumption.

Figure 11 shows the impulse responses after a shock to the government debt target in a Pure Currency Union scenario when deleveraging is achieved with government transfers. We can notice that if the international trade elasticity is low (dotted blue line) or the labour supply elasticity is low (dashed green line), consumption moves more, but affects less movements in net exports, either because it responds less to movements in the terms of trade, when the international trade elasticity is lower, or because the terms of trade move less not allowing consumption to be smoothed, when there is a lower labour supply elasticity in both countries. As a consequence the response of GDP is smoother in both cases, while inflation and the interest rate move more. Therefore, we conclude that aggregate demand is less sensitive to international adjustment if the flattening of the labour supply curve happens through a lower elasticity of labour supply compared to a lower international trade elasticity (or a higher home bias).