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# Government Debt Deleveraging in the EMU

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**Alexandre Lucas Cole**

(co-authored with Chiara Guerello and Guido Traficante)

*LUISS Guido Carli (Rome)*

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- 1 Introduction
- 2 A Two-Country Currency Union Model
- 3 Calibration
- 4 Numerical Simulations
- 5 Welfare Analysis
- 6 Conclusions and Possible Extensions

# Introduction - Motivation

- After the recent global crisis, there has been a great discussion on the future of European economic integration and on **the role of the austerity measures imposed by sovereign debt reduction**.
- Given a situation of high government debt in most EMU countries and a **request by the European Commission to reduce government debt positions to 60% of GDP**, finding the best way and timing for deleveraging is an important issue.
- **We evaluate the stabilization properties and welfare implications of different deleveraging schemes and instruments**, under alternative scenarios for fiscal policy coordination, bringing to policy conclusions for the proper government debt management in a Currency Union.

# Introduction - Strategy and Main Results

We build a Two-Country DSGE model of a Currency Union, with a **debt-elastic government bond spread** and **incomplete international financial markets**.

Our main findings are:

- **Coordinating by reducing international demand imbalances** and creating some form of fiscal union across countries provides more stabilization when reducing government debt.
- **Using distortionary taxes** is the most stabilizing way to reduce government debt.
- By **reducing government debt more gradually over time** one can achieve greater stabilization.
- **Government debt should be reduced less during recessions and liquidity traps.**

We follow two strands of literature:

- **Open Economy – Currency Union:** Silveira (2006), Galí (2009), Ferrero (2009), Hjortsø (2016), Cole, Guerello and Traficante (2016).
- **Debt Deleveraging:** Coenen, Mohr and Straub (2008), Forni, Gerali and Pisani (2010), Cogan et al. (2013), Romei (2015).

We focus on:

- **Public debt reduction rule** and deleveraging shocks in the Periphery.
- **Targeting rules for fiscal policy**, to allow governments to coordinate.

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# Households

Each Household in country H seeks to **maximize the present-value utility**:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[ \frac{(C_t^i)^{1-\sigma} - 1}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \right] \quad (2.1)$$

**subject to the following sequence of budget constraints:**

$$\int_0^h P_{H,t}(j) C_{H,t}^i(j) dj + \int_h^1 P_{F,t}(j) C_{F,t}^i(j) dj + D_t^i + B_{H,t}^i + B_{F,t}^i \\ \leq \frac{D_{t-1}^i}{Q_{t-1,t}} + B_{H,t-1}^i(1+i_{t-1}) + B_{F,t-1}^i(1+i_{t-1}^*)(1-\delta_{t-1}) + (1-\tau_t^w)W_t N_t^i + T_t^i + \Gamma_t^i + \mathcal{I}_t^i \quad (2.2)$$

where  $B_{H,t}^i$  are government bonds issued by country H which yield a return given by  $i_{t-1}$ , while  $B_{F,t}^i$  are government bonds issued by country F which yield a return  $i_{t-1}^*$ , while  $\delta_t \in [0, 1]$  is a **transaction cost for households in country H on purchases of government bonds issued by country F**, given by:

$$\delta_t \equiv (1 - \rho_\delta) \delta^B \left( \frac{B_{t-1}^{*G}}{P_{H,t-1}^* Y_{t-1}^*} - \frac{B_{t-1}^{*G}}{P_H^* Y^*} \right) + \rho_\delta \delta_{t-1} \quad (2.3)$$

where  $\frac{B_{t-1}^{*G}}{P_{H,t-1}^* Y_{t-1}^*}$  is the overall real government debt-to-GDP for country F.

[More Details](#)

# International Assumptions

$C_t^i$  is a **composite index for private consumption** defined by:

$$C_t^i \equiv \left[ (1 - \alpha)^{\frac{1}{\eta}} (C_{H,t}^i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t}^i)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (2.4)$$

If  $1 - \alpha > h$  there is **home bias** in consumption in country H, because the **share of consumption** of domestic goods is **greater than the share of production** of domestic goods.

- $\alpha \in [0, 1]$  is a measure of **openness of the economy to international trade**.
- $(1 - \alpha)$  is a measure of the degree of **home bias** in consumption.

The **terms of trade** are defined as the price of foreign goods in terms of home goods:

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} \quad (2.5)$$

Although deviations from *Purchasing Power Parity (PPP)* may arise because of home bias in consumption, we assume that the **Law of One Price (LOP)** holds for every single good  $j$ :

$$P_{H,t}(j) = P_{F,t}^*(j) \quad \text{and} \quad P_{F,t}(j) = P_{H,t}^*(j) \quad (2.6)$$

# Incomplete International Financial Markets

Households can trade a complete set of one-period state-contingent claims only within their own country. **Households in country H can purchase one-period bonds issued by both countries' governments**, while **households in country F can only purchase one-period bonds issued by their own country's government**.

From the no-arbitrage condition on bonds for households in country H:

$$\frac{1}{(1+i_t^*)(1-\delta_t)} = \frac{1}{1+i_t} = E_t\{Q_{t,t+1}\} = \beta E_t\left\{\frac{\xi_{t+1}}{\xi_t} \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1}{\Pi_{t+1}}\right\} \quad (2.7)$$

which shows **there is no full international risk-sharing**.

**The interest rate paid on government bonds issued by country F** is then given by:

$$1+i_t^* = \frac{1+i_t}{1-\delta_t} \quad (2.8)$$

and is increasing in the transaction cost  $\delta_t$ , or in the government bond spread  $(1+i_t^*)\delta_t$ , other than increasing in the interest rate set by the central bank and paid on government bonds issued by country H,  $i_t$ .

In country H there is a continuum of Firms indexed by  $j \in [0, h)$ , each producing a differentiated good with the same technology represented by the following **production function**:

$$Y_t(j) = A_t N_t(j) \quad (2.9)$$

where  $A_t$  represents the country-specific **level of technology**.

Firm  $j$ 's **period t profits net of taxes** in country H are given by:

$$\Gamma_t(j) = (1 - \tau_t^s) P_{H,t}(j) Y_t(j) - W_t N_t(j) \quad (2.10)$$

where  $\tau_t^s$  is the marginal **tax rate on firm sales**.

- Following Calvo (1983), **each firm may reset its price with probability  $1 - \theta$  in any given period**.
- The **average duration of a price** is given by  $(1 - \theta)^{-1}$
- $\theta$  can be seen as a natural **index of price stickiness** for country H.
- The index of **price stickiness in the two countries can differ**:  $\theta \neq \theta^*$

Monetary policy follows an **Inflation Targeting regime** of the kind:

$$\beta(1 + i_t) = \left( \frac{\Pi_t^U}{\Pi^U} \right)^{\phi_\pi(1-\rho_i)} [\beta(1 + i_{t-1})]^{\rho_i} \quad \Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \quad (2.11)$$

where  $\phi_\pi$  represents the **responsiveness of the interest rate to inflation** and  $\rho_i$  is a measure of the persistence of the interest rate.

We also consider **the case of the Zero Lower Bound constraint**:

$$i_t = \max \{ \tilde{i}_t, 0 \} \quad \beta(1 + \tilde{i}_t) = \left( \frac{\Pi_t^U}{\Pi^U} \right)^{\phi_\pi(1-\rho_i)} [\beta(1 + \tilde{i}_{t-1})]^{\rho_i} \quad (2.12)$$

where  $\tilde{i}_t$  is the shadow interest rate, which is the unconstrained level of the nominal interest rate.

In country H **the government finances a stream of public consumption  $G_t$  and transfers  $\tilde{T}_t$  subject to the following sequence of budget constraints:**

$$G_t + \tilde{T}_t + i_{t-1} \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad (2.13)$$

- $\tilde{B}_t^G$  is overall **real government debt** in country H
- **the left hand side represents current government expenditure and interest payments on outstanding debt.**
- **the right hand side represents government financing of that expenditure** through taxes and the possible variation of government debt.

**Government consumption is characterized by complete home bias.**

# Pure Currency Union - Distortionary Tax Scenario

Fiscal policy **chooses government consumption to stabilize the output gap countercyclically**:

$$\frac{G_t^*}{G^*} = \left( \frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_g^*)} \left( \frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (2.14)$$

while keeping real transfers constant and **varying equally the tax rates on labour income and firm sales to deleverage its government debt and to finance the remaining government expenditure**:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left( \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \quad \tilde{\tau}_t^* = \tilde{\tau}^* \quad (2.15)$$

$$\tau_t^{*W} - \tau^{*W} = \tau_t^{*S} - \tau^{*S} \quad (2.16)$$

where  $\psi_y^* \geq 0$  represents the **responsiveness of government consumption to variations of the output gap** and  $\gamma_t^* \in [0, 1]$  is the **desired share of reduction per period of the excess real government debt with respect to steady state**.

# Pure Currency Union - Transfer Scenario

Fiscal policy **chooses government consumption to stabilize the output gap countercyclically:**

$$\frac{G_t^*}{G^*} = \left( \frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_g^*)} \left( \frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (2.17)$$

while **using real transfers  $\tilde{T}_t^*$  to deleverage its government debt:**

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left( \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} \right) \quad (2.18)$$

and **varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure:**

$$\tau_t^{*W} - \tau^{*W} = \tau_t^{*S} - \tau^{*S} \quad (\tau_t^{*S} + \tau_t^{*W} MC_t^* d_t^*) Y_t^* - (\tau^{*S} + \tau^{*W} MC^*) Y^* = G_t^* - G^* \quad (2.19)$$

# Coordinated Currency Union - Transfer Scenario

Fiscal policy **chooses government consumption to stabilize its real net exports gap procyclically:**

$$\frac{G_t^*}{G^*} = \left( \frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_g^*)} \left( \frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (2.20)$$

while **using real transfers  $\tilde{T}_t^*$  to deleverage its government debt:**

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left( \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \quad (2.21)$$

and **varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure:**

$$\tau_t^{*w} - \tau^{*w} = \tau_t^{*s} - \tau^{*s} \quad (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) Y_t^* - (\tau^{*s} + \tau^{*w} MC^*) Y^* = G_t^* - G^* \quad (2.22)$$

where  $\psi_{nx}^* \geq 0$  represents the **responsiveness of government consumption to variations of the output gap** and  $\gamma_t^* \in [0, 1]$  is the **desired share of reduction per period of the excess real government debt with respect to steady state.**

A Full Fiscal Union **uses local government spending to manage fiscal policy at the union level with a consolidated budget constraint:**

$$P_{H,t}G_t + P_{H,t}^*G_t^* + T_t + T_t^* + B_{t-1}^G(1 + i_{t-1}) + B_{t-1}^{*G} \frac{1 + i_{t-1}}{1 - \delta_{t-1}} =$$
$$B_t^G + B_t^{*G} + \tau_t^S P_{H,t} Y_t + \tau_t^{*S} P_{H,t}^* Y_t^* + \tau_t^W W_t N_t + \tau_t^{*W} W_t^* N_t^* \quad (2.23)$$

In this case government debt will be aggregated across countries and both countries will contribute to the deleveraging of government debt. Nonetheless, **given that financial markets are still incomplete, there continue to be two separate government bonds for the two countries, which pay different interest rates and so have different bond yields.**

# Full Fiscal Union - Transfer Scenario

Union-wide fiscal policy **chooses government consumption in each country to stabilize its real net exports gap procyclically:**

$$\frac{G_t^*}{G^*} = \left( \frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_g^*)} \left( \frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (2.24)$$

while **using real transfers equally in both countries to deleverage the government debt of country F**, while country H maintains its government debt constant:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left( \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad \tilde{T}_t - \tilde{T} = \tilde{T}_t^* - \tilde{T}^* \quad (2.25)$$

and **varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure:**

$$\tau_t^W - \tau^W = \tau_t^S - \tau^S \quad \tau_t^{*W} - \tau^{*W} = \tau_t^W - \tau^W \quad \tau_t^{*S} - \tau^{*S} = \tau_t^S - \tau^S \quad (2.26)$$

$$(\tau_t^S + \tau_t^W MC_t d_t) Y_t + (\tau_t^{*S} + \tau_t^{*W} MC_t^* d_t^*) S_t Y_t^* - (\tau^S + \tau^W MC) Y - (\tau^{*S} + \tau^{*W} MC^*) Y^* = G_t + G_t^* - G - G^* \quad (2.27)$$

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# Calibration - Structure

Following Ferrero (2009), we consider the top 5 Eurozone countries, which account for more than 80% of Eurozone GDP and we divide them into:

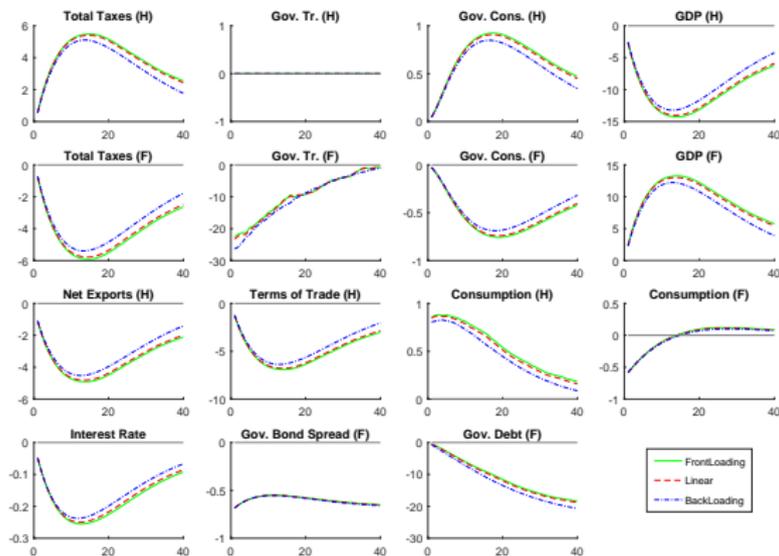
- 1 **Country F**, the periphery (namely **France, Italy, Spain and The Netherlands**)
  - 2 **Country H**, the core (namely **Germany**)
- **The annualized steady state value of government debt-to-GDP in both countries is set to roughly 60%**, as stated in the Maastricht Treaty.
  - In the simulations, **country F starts with a higher level of government debt-to-GDP, equal to roughly 80%**, in line with the average level of government debt-to-GDP for France, Italy, Spain and The Netherlands.
  - **For every ten percentage points increase in government debt-to-GDP the government bond spread increases by 9 percentage points**, according to which we set  $\delta^B = 0.009$ .
  - **The desired fraction of reduction of excess government debt is set to  $\gamma_t^* = 0.05$  for country F, corresponding to a 5% yearly reduction**, to comply with the Debt Brake Rule in the Fiscal Compact, and to  $\gamma_t = 0$  for country H, as **only country F needs to deleverage**.

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# Deleveraging Schemes - Pure Currency Union

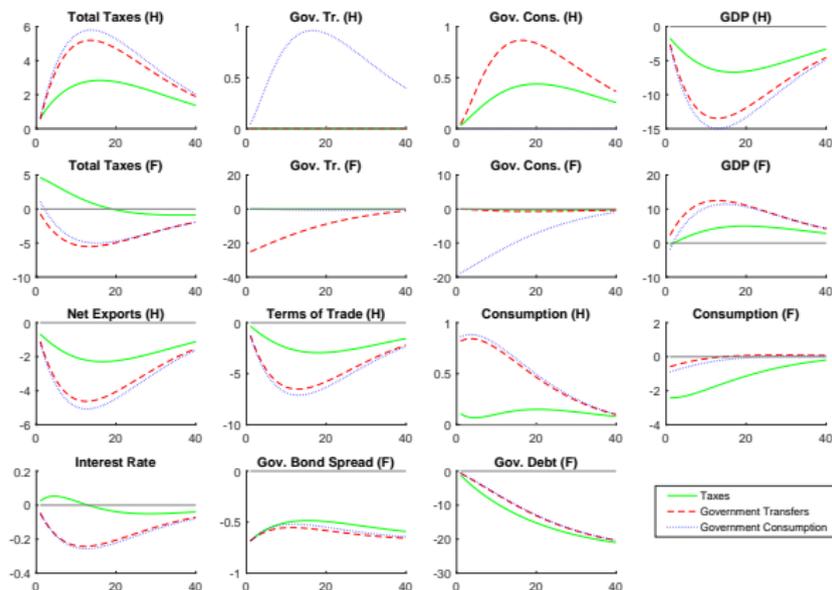
Deleveraging with Transfers in Pure Currency Union



Here we compare: Frontloading ( $\gamma_t$  from 13% to 0.1% in 10 years), Backloading ( $\gamma_t$  from 1% to 10% in 10 years) and Linear ( $\gamma_t$  constant at 5%). [More Details](#)

# Instruments for Deleveraging - Pure Currency Union

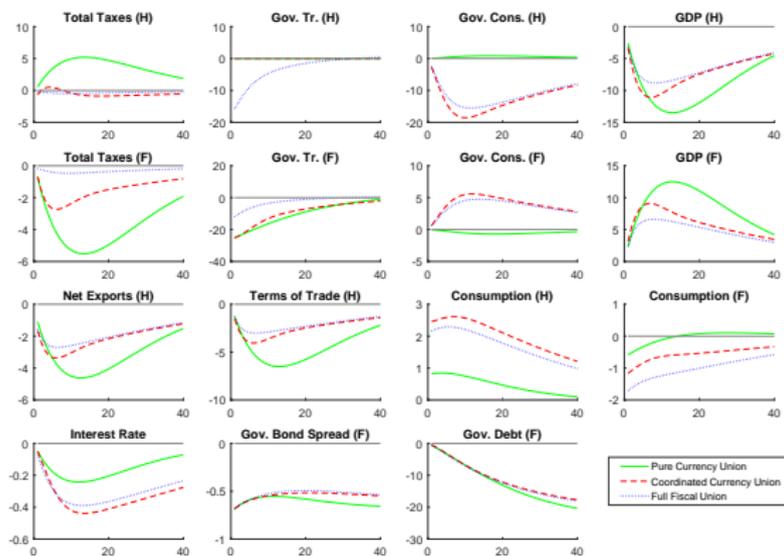
Deleveraging in Pure Currency Union - Deleveraging Shock in Country F



Here we compare different fiscal instruments: taxes, government consumption and government transfers. Full Fiscal Union

# Coordination of Deleveraging with Government Transfers

Deleveraging with Transfers - Deleveraging Shock in Country F

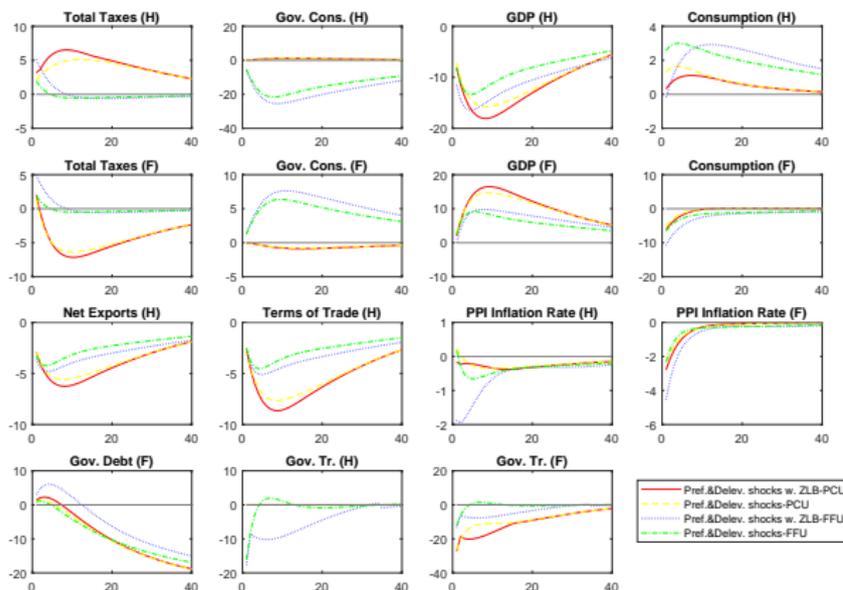


Here we compare different degrees of coordination: Pure Currency Union, Coordinated Currency Union, and Full Fiscal Union.

Deleveraging with Taxes

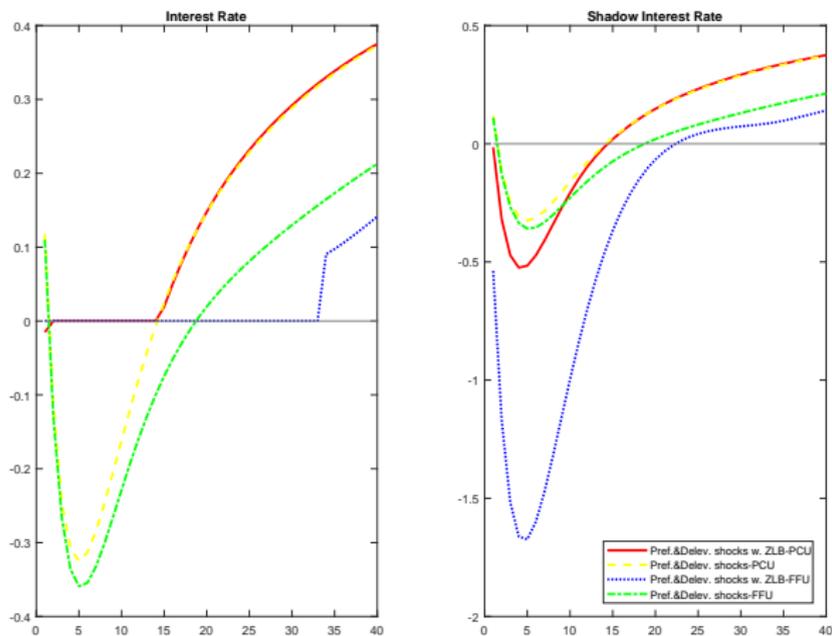
# Coordination of Deleveraging at the ZLB

Comparison of Deleveraging with Transfers with ZLB



Here we compare Pure Currency Union and Full Fiscal Union with and without the ZLB constraint.

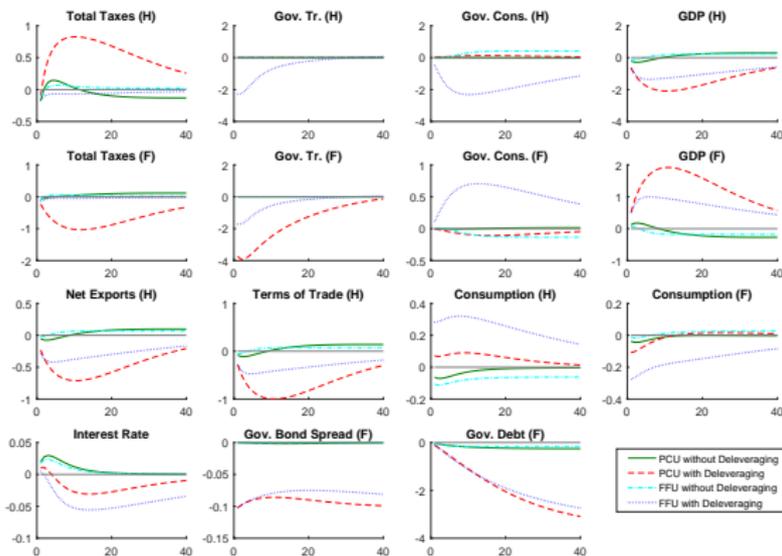
# Duration of the Liquidity Trap



Here we show the nominal interest rate and the shadow interest rate from which one can see the duration of the liquidity trap.

# Net Shocks from Deleveraging with Government Transfers

Net Shock with Government Transfers - Technology Shock in Country H



Here we compare the response to a negative technology shock in country H when country F is deleveraging and when it is not (net shocks).

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# Welfare Costs of Deleveraging Scenarios by Instrument

**We compare the stabilization properties** of the fiscal policy scenarios and of the deleveraging instruments **by means of an ad hoc Loss Function**. **Here we compare the welfare costs for the three scenarios for fiscal policy coordination.**

**Table:** Welfare Costs: Comparison of Fiscal Scenarios by Instrument

Welfare Costs based on ad hoc loss function			
Fiscal Instrument: Government Consumption			
	Country H	Country F	Average
PCU	216.3%	160.7%	188.1%
CCU	9.33%	9.38%	9.36%
FFU *	0%	0%	0%
Fiscal Instrument: Government Transfers			
	Country H	Country F	Average
PCU	93.55%	196.9%	140.7%
CCU	22.99%	49.51%	35.09%
FFU*	0%	0%	0%
Fiscal Instrument: Taxes on Sales and Wages			
	Country H	Country F	Average
PCU	25.02%	82.64%	45.19%
CCU *	0%	0%	0%
FFU	50.20%	62.72%	54.58%

Welfare Costs are computed as  $\frac{Loss_a - Loss_b}{Loss_b}$ , with  $b$  the scenario featuring the lowest loss

for the selected fiscal instrument (indicated with \*)

# Welfare Costs of Deleveraging Instruments by Scenario

Here we compare the welfare costs of using a specific fiscal instrument for deleveraging in each of the three scenarios for fiscal policy coordination.

Table: Welfare Costs: Comparison of Fiscal Instruments by Scenario

Welfare Costs based on ad hoc loss function			
Fiscal Scenario: Pure Currency Union			
	Country H	Country F	Average
Gov. Cons.	292.3%	323.3%	305.9%
Gov. Tr.	211.0%	409.1%	298.2%
Taxes*	0%	0%	0%
Fiscal Scenario: Coordinated Currency Union			
	Country H	Country F	Average
Gov. Cons.	69.53%	224.3%	123.7%
Gov. Tr.	147.1%	368.2%	224.5%
Taxes*	0%	0%	0%
Fiscal Scenario: Full Fiscal Union			
	Country H	Country F	Average
Gov. Cons.	3.23%	82.20%	32.33%
Gov. Tr.	33.75%	92.46%	55.38%
Taxes*	0%	0%	0%

Welfare Costs are computed as  $\frac{Loss_a - Loss_b}{Loss_b}$ , with  $b$  the instrument featuring the lowest loss for the selected fiscal scenario (indicated with \*)

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# Conclusions and Possible Extensions

- **Coordinating on the net exports gap** and (to a minor extent) consolidating budget constraints when deleveraging provides more stabilization.
- **Taxes are a better instrument for deleveraging** compared to government consumption or transfers.
- By **backloading the deleveraging process** one can achieve greater stabilization over time: timing of deleveraging matters!
- **Deleveraging government debt amplifies negative technology shocks.**
- In presence of the ZLB deflationary pressures are stronger and **when deleveraging the liquidity trap lasts longer.**

## Possible Extensions:

- Different coordination strategies for national fiscal policies can be imagined.
- A more complex structure of international financial markets might change the amount of private risk-sharing across countries and the international transmission of shocks.
- Distributional consequences of fiscal consolidations may matter, with government transfers used to reduce inequalities.

**Thank you for your attention!**



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## 7 More Details

**The financial intermediaries**, owned by the households in country H, **earn profits on all the internationally traded bonds**  $B_{F,t-1}^i$  **by collecting savings from households in country H at the interest rate set by the central bank**  $i_{t-1}$  **and lending to the government in country F at the interest rate paid on its government bonds**  $i_{t-1}^*$ . The aggregate profits of these financial intermediaries are given by:

$$\mathcal{I}_t \equiv B_{F,t-1} \left[ (1 + i_{t-1}^*) - (1 + i_{t-1}^*)(1 - \delta_{t-1}) \right] = B_{F,t-1} (1 + i_{t-1}^*) \delta_{t-1} \quad (8.1)$$

where  $B_{F,t-1} \equiv \int_0^h B_{F,t-1}^i di$  are aggregate bonds issued by the government in country F and purchased by households in country H and where **the government bond spread for country F, on which financial intermediaries make profits, is given by**  $(1 + i_{t-1}^*) \delta_{t-1}$ .

# Net Exports and the Balance of Payments

**Net Exports** for country H are given by:

$$NX_t \equiv P_{H,t}Y_t - P_tC_t - P_{H,t}G_t \quad (8.2)$$

**Net Foreign Assets** for country H are given by:

$$NFA_t \equiv D_t + B_t - B_t^G \quad (8.3)$$

The **Balance of Payments** for country H is given by:

$$BP_t \equiv NX_t + i_{t-1}NFA_{t-1} \quad (8.4)$$

so that **Net Foreign Assets for country H** evolve according to:

$$NFA_t = (1 + i_{t-1})NFA_{t-1} + NX_t = NFA_{t-1} + BP_t \quad (8.5)$$

Back to [International Assumptions](#)

A firm in country H re-optimizing in period  $t$  will **choose the price  $\bar{P}_{H,t}$  that maximizes the current market value of the profits generated while that price remains effective**, formally solving the problem:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \{ Q_{t,t+k} Y_{t+k|t}(j) [(1 - \tau_{t+k}^s) \bar{P}_{H,t} - MC_{t+k}^n] \} \quad (8.6)$$

where  $Q_{t,t+k}$  is the **household's stochastic discount factor**.

One can then express the **optimal price chosen by firms** in country H as a function of only aggregate variables:

$$\bar{P}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} MC_{t+k}^n \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} (1 - \tau_{t+k}^s) \right\}} \quad (8.7)$$

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# Pure Currency Union - Consumption Scenario

Fiscal policy **chooses real transfers to stabilize the output gap countercyclically**, while following in part an exogenous process:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left( \frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_t^*)} \left( \frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} \quad (8.8)$$

while **using government consumption  $G_t^*$  to deleverage its government debt**:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left( \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} \right) \quad (8.9)$$

and **varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure**:

$$\tau_t^{*W} - \tau^{*W} = \tau_t^{*S} - \tau^{*S} \quad (\tau_t^{*S} + \tau_t^{*W} MC_t^* d_t^*) Y_t^* - (\tau^{*S} + \tau^{*W} MC^*) Y^* = \tilde{T}_t^* - \tilde{T}^* \quad (8.10)$$

Back to [Pure Currency Union - Transfer Scenario](#)

# Coordinated Currency Union - Consumption Scenario

Fiscal policy **chooses real transfers to stabilize its real net exports gap procyclically**, while following in part an exogenous process:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left( \frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_t^*)} \left( \frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} \quad (8.11)$$

while **using government consumption  $G_t^*$  to deleverage its government debt**:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left( \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} \right) \quad (8.12)$$

and **varying equally the tax rates on labour income and firm sales to finance the remaining government expenditure**:

$$\tau_t^{*W} - \tau^{*W} = \tau_t^{*S} - \tau^{*S} \quad (\tau_t^{*S} + \tau_t^{*W} MC_t^* d_t^*) Y_t^* - (\tau^{*S} + \tau^{*W} MC^*) Y^* = \tilde{T}_t^* - \tilde{T}^* \quad (8.13)$$

where  $\psi_{nx}^* \geq 0$  represents the **responsiveness of government consumption to variations of the output gap** and  $\gamma_t^* \in [0, 1]$  is the **desired share of reduction per period of the excess real government debt with respect to steady state**.

# Coordinated Currency Union - Distortionary Tax Scenario

Fiscal policy **chooses government consumption to stabilize its real net exports gap procyclically:**

$$\frac{G_t^*}{G^*} = \left( \frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_g^*)} \left( \frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (8.14)$$

while keeping real transfers constant and **varying equally the tax rates on labour income and firm sales to deleverage its government debt and to finance the remaining government expenditure:**

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left( \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \quad \tilde{\tau}_t^* = \tilde{\tau}^* \quad (8.15)$$

$$\tau_t^{*W} - \tau^{*W} = \tau_t^{*S} - \tau^{*S} \quad (8.16)$$

where  $\psi_{nx}^* \geq 0$  represents the **responsiveness of government consumption to variations of the output gap** and  $\gamma_t^* \in [0, 1]$  is the **desired share of reduction per period of the excess real government debt with respect to steady state.**

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Transfer Scenario

# Full Fiscal Union - Consumption Scenario

Union-wide fiscal policy **chooses real transfers in each country to stabilize its real net exports gap procyclically**, while following in part an exogenous process:

$$\frac{\tilde{T}_t^*}{\bar{T}^*} = \left( \frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_t^*)} \left( \frac{\tilde{T}_{t-1}^*}{\bar{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} \quad (8.17)$$

while **using government consumption equally in both countries to deleverage the government debt of country F**, while country H maintains its government debt constant:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left( \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad G_t - G = G_t^* - G^* \quad (8.18)$$

and **varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure**:

$$\tau_t^W - \tau^W = \tau_t^S - \tau^S \quad \tau_t^{*W} - \tau^{*W} = \tau_t^W - \tau^W \quad \tau_t^{*S} - \tau^{*S} = \tau_t^S - \tau^S \quad (8.19)$$

$$(\tau_t^S + \tau_t^W MC_t d_t) Y_t + (\tau_t^{*S} + \tau_t^{*W} MC_t^* d_t^*) S_t Y_t^* - (\tau^S + \tau^W MC) Y - (\tau^{*S} + \tau^{*W} MC^*) Y^* = \tilde{T}_t + \tilde{T}_t^* - \tilde{T} - \tilde{T}^* \quad (8.20)$$

# Full Fiscal Union - Distortionary Tax Scenario

Union-wide fiscal policy **chooses government consumption in each country to stabilize its real net exports gap procyclically:**

$$\frac{G_t^*}{G^*} = \left( \frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_g^*)} \left( \frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (8.21)$$

while keeping real transfers constant and **varying equally the tax rates on labour income and firm sales to deleverage the government debt of country F**, while country H maintains its government debt constant:

$$\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} = \gamma_t^* \left( \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}^{*G} \right) \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad \tilde{T}_t - \tilde{T} = \tilde{T}_t^* - \tilde{T}^* \quad (8.22)$$

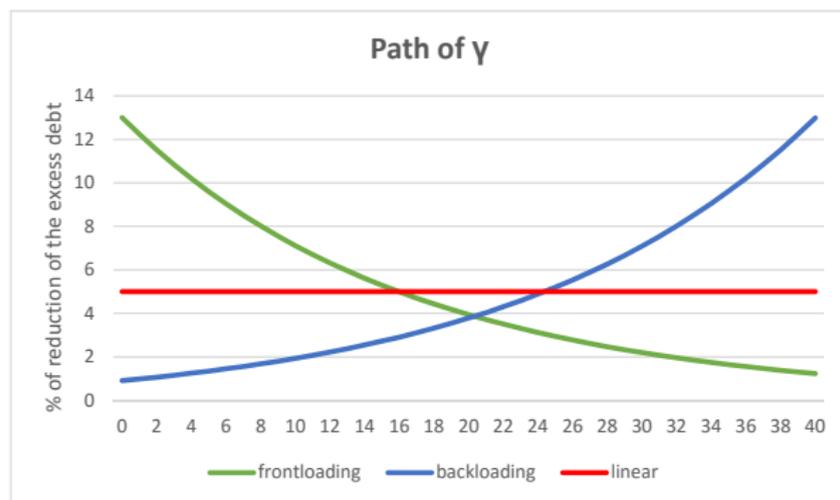
and also **varying equally across countries the tax rates on labour income and firm sales to finance the remaining government expenditure:**

$$\tau_t^W - \tau^W = \tau_t^S - \tau^S \quad \tau_t^{*W} - \tau^{*W} = \tau_t^W - \tau^W \quad \tau_t^{*S} - \tau^{*S} = \tau_t^S - \tau^S \quad (8.23)$$

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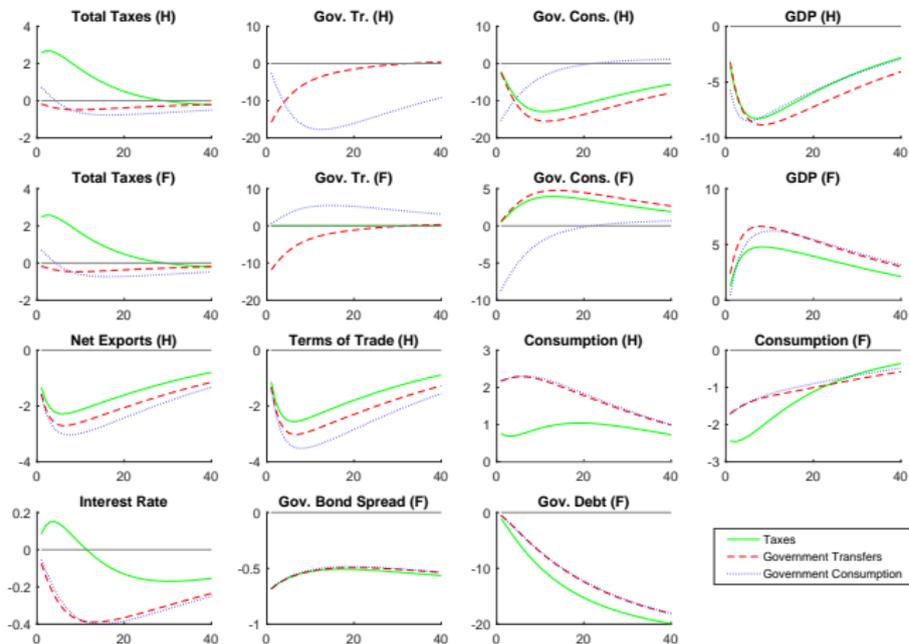
# Deleveraging Paths

The **three deleveraging paths over time** are shown in terms of the percent reduction of excess government debt:



# Instruments for Deleveraging - Full Fiscal Union

## Deleveraging in Full Fiscal Union - Deleveraging Shock in Country F



# Coordination of Deleveraging with Taxes

## Deleveraging with Taxes - Deleveraging Shock in Country F

